PART VI

COMPOUND CURVES

Any simple circular curve used in a highway, railway or canal presents problems.

At the P.C. there is an instantaneous change of direction and the magnitude of this change increases as the radius decreases. In the case of a highway or railway, a vehicle or a train cannot change direction instantaneously. In the case of the vehicle, it does not follow a truly circular path in the first part of the curve. In the case of the train it imposes forces on the rails which may cause them to shift. The magnitude of the forces will increase as R decreases assuming that the velocity remains constant.

There is also the problem of super-elevating or "banking" a simple circular curve. The greater the design speed the greater will be the need for super-elevation. Theoretically, there should be no superelevation on one side of the P.C. and there should be the required superelevation on the other, with no transition in between. Because such a situation is impractical we have to compromise and perhaps begin our superelevation before the P.C. is reached so that the first part of the curve has, at least, part of the required superelevation. The situation becomes worse as R is decreased, for a constant design speed.

Obviously what has been said in the preceding two paragraphs applies at the P.T. as well.

On canals, too rapid a change in direction could lead to erosion problems.

Because the magnitude of the problems of change of direction and super-elevation increase as R decreases we can, at least partially, relieve the problem by combining arcs of simple curves of different radius as shown.

The curve shown in Fig.1 consists of two outer arcs, both having the same long radius and the same central angle. The central portion has a short radius. In this case the curve is symmetrical about the dotted line and is called a Symmetrically Compounded Curve.

The long radius portion between the P.C. and the P.C.C. (Point of Compound Curvature) allows for a more gradual change in direction and a more gradual (and smoother) application of superelevation so that the short radius portion will be safe. The second long radius portion (from the second P.C.C. to the P.T.) allows for a gradual change back to straight-line conditions beyond the P.T.
We can have a multi-centred curve as illustrated below.

This is a multi-centered curve which happens to be symmetrical about the dotted line.

In terms of riding and driving qualities it would be an improvement over the symmetrically-compounded curve in Figure 1.

Later we will discuss a better solution to the problem—the so-called transition spiral.

Figure 2

There are many applications of multi-centered curves which are not symmetrical—particularly in various types of grade-separation structures.
Calculate the sub-tangent distance, $T$, for the symmetrically-compounded curve shown above.

**Solution**

In this case we can make use of symmetry to set up a simple solution. We can treat $AOB$ and $BOC$ each as separate self-contained curves. Through $B$ we can draw a line $V_1BV_2$, perpendicular to $BO$, which is tangent to both curves.
AV₁ = V₁B = V₁
BV₂ = V₂C = V₂
V₁V₂ = T₁ + T₂
Angle AVV₂ = \frac{180°-\Delta}{2} = 60°

Angle VV₁V₂ = \Delta₁ = 15°

By deduction, Angle V₁V₂V = 180° - 60° - 15° = 105°

T₁ = 500 \times \tan 7°30' = 65.82
T₂ = 300 \times \tan 15° = 80.39
V₁V₂ = 146.21

We can use the sin law to solve for V₁V in the triangle VVV₁V₂

\frac{146.21}{\sin 60°} = \frac{V₁V}{\sin 105°}

V₁V = \frac{146.21 \times 0.96593}{0.86603} = 163.08

AV = VD = 163.08 + 65.82 = 228.90 m

\boxed{T = 228.90}

We can now try an example where the curve is not symmetrical.
2. Example Two

In this case AV does not equal DV.

Figure 5

Figure 6
\[ AV_1 = V_1B = T_1 = 400 \quad \text{Tan} 10^\circ = 70.53 \]
\[ BV_2 = V_2C = T_2 = 300 \quad \text{Tan} 10^\circ = 52.89 \]
\[ CV_3 = V_3D = T_3 = 200 \quad \text{Tan} 10^\circ = 35.27 \]

In Figure 6

Angle \( V_1V_2V_3 = 120^\circ \)
\( VV_1V_2 = 20^\circ \)
\( V_1V_2V_3 = 200^\circ \)
\( V_2V_3V = \frac{20^\circ}{360^\circ} \)

\[ AV_1 = T_1 = 70.53 \]
\[ V_1V_2 = T_1 + T_2 = 70.53 + 52.89 = 123.43 \]
\[ V_2V_3 = T_2 + T_3 = 52.89 + 35.37 = 88.16 \]
\[ V_3D = T_3 = 35.27 \]

We must now calculate the lengths \( AV \) and \( VD \) by the method of latitudes and departures.

The approach to all such problems will be the same. As the number of different arcs is increased the only change will be the length of the latitudes and departures problem.

<table>
<thead>
<tr>
<th>Side</th>
<th>Traverse Data</th>
<th>Functions</th>
<th>Latitudes</th>
<th>Departures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Length Bearing</td>
<td>Cos</td>
<td>Sin</td>
<td>+</td>
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<tr>
<td>( VV_1 )</td>
<td>- Ass.West</td>
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<tr>
<td>( V_1V_2 )</td>
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<td>( V_2V_3 )</td>
<td>88.16 S50°E</td>
<td>.64279</td>
<td>.76604</td>
<td>56.67</td>
</tr>
<tr>
<td>( V_3V )</td>
<td>114.19 N30°W</td>
<td>.86603</td>
<td>.30000</td>
<td>98.88 ( (1) )</td>
</tr>
</tbody>
</table>
1. \( \Sigma \text{Southings} = 98.88 = \Sigma \text{Northings} \)
   
   All Northings are in \( V_3V = \frac{98.88}{\cos 30^0} \) 

2. \( V_3V \times \cos 30^0 = 98.88 \)
   
   \[ V_3V = \frac{98.88}{\cos 30^0} = \frac{98.88}{0.86603} = 114.19 \]  

3. Dep. in \( V_3V = 114.19 \times 0.5000 = 57.09 \)  

4. \( \Sigma \text{Eastings} = 183.52 = \Sigma \text{Westings} \)
   
   Balance of Westings is \( VV_1 \)
   
   \[ VV_1 = 183.52 - 57.09 = 126.43 \]  

   Back Tangent = \( AV = VV_1 + T_1 \)
   
   \[ AV = 126.43 + 70.53 \]
   
   \[ AV = 196.96 \]  

   Forward Tangent = \( VD = VV_3 + T_3 \)
   
   \[ VD = 149.45 \]  

   Note that, in any compound curve, the curve must be run in sections, the instrument occupying the beginning or end of each section of different radius.

   It should also be noted that it is necessary to put in control independent of the curve in order to isolate any errors that may be made. In the case of the assymetrical compound curve in Figures 5 and 6 it is highly desirable (in fact, essential) that points \( V_1, B, V_2, C \) and \( V_3 \) be located before one begins running the curve.

3. **Example Three**

   Two tangents intersect at \( \Delta = 70^0, \) P.I. = Stn 32 + 51.82. Set up the field notes for the following compound curve (symmetrical).

   1st curve \( \Delta = 20^0, D = 8^0R \) (20 m chords)  
   2nd curve \( \Delta = 30^0, D = 12^0R \) (10 m chords)  
   3rd curve \( \Delta = 20^0, D = 8^0R \) (20 m chords)
Solution

1st curve \( R_1 = \frac{20}{2 \sin \left(\frac{8}{2}\right)} = 143.36 \text{ m} \)

\[ T_1 = \frac{R_1 \tan \Delta_1}{2} = (143.36) \tan \left(\frac{20}{2}\right) = 25.28 \text{ m} \]

2nd curve \( R_2 = \frac{10}{2 \sin \left(\frac{12}{2}\right)} = 47.83 \text{ m} \)

\[ T_2 = \frac{R_2 \tan \Delta_2}{2} = (47.83) \tan \left(\frac{30}{2}\right) = 12.82 \text{ m} \]

3rd curve = same as 1st curve
\[ AV_1 = T_1 = 25.28 \text{m} \]

In \( \Delta V_1V_2V \)

\[ V_1V_2 = T_1 + T_2 = 25.28 + 12.82 \]

\[ V_1V_2 = 38.10 \]

\[ \angle V_1V_2 = (180-70)/2 = 55^\circ \]

\[ \angle VV_1V_2 = \Delta_1 = 20^\circ \]

\[ \angle V_1V_2V = 180-75 = 105^\circ \]

Using sine law

\[ \frac{V_1V}{\sin 105^\circ} = \frac{38.10}{\sin 55^\circ} \]

\[ V_1V = 44.93 \text{ m} \]

\[ \therefore \text{Back tangent} = AV_1 + V_1V = 25.28 + 44.93 \]

\[ \text{B. T.} = 70.21 \text{ m} \]

Since curve is symmetrical the forward tangent = back tangent = 70.21 m

\[ L_{c_1} = \frac{\Delta}{D} \times 20 = \frac{20}{8} \times 20 = 50.00 \text{ m} \]

\[ L_{c_2} = \frac{\Delta}{D} \times 10 = \frac{30}{12} \times 10 = 25.00 \text{ m} \]

\[ L_{c_3} = L_{c_1} = 50.00 \text{ m} \]

\[ L_{cc} = 125.00 \text{ m} \]
### Figure 9

<table>
<thead>
<tr>
<th>Station</th>
<th>Def Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
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<tr>
<td></td>
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<tr>
<td>P.I. = Stn</td>
<td>32 + 51.82 Check</td>
</tr>
<tr>
<td>- B.T.</td>
<td>00 + 70.21  P.C. 31 + 81.51</td>
</tr>
<tr>
<td>P.C.</td>
<td>31 + 81.61 + Lcc 1 + 25.00</td>
</tr>
<tr>
<td>+ Lc1</td>
<td>00 + 50.00 P.T. 33 + 06.61</td>
</tr>
<tr>
<td>P.C.C.1</td>
<td>32 + 31.61</td>
</tr>
<tr>
<td>+ Lc2</td>
<td>00 + 25.00</td>
</tr>
<tr>
<td>P.C.C.2</td>
<td>32 + 56.61</td>
</tr>
<tr>
<td>+ Lc3</td>
<td>00 + 50.00</td>
</tr>
<tr>
<td>P.T.</td>
<td>33 + 06.61</td>
</tr>
</tbody>
</table>

### Angle Def

\[
\Delta_T = 20 + 30 + 20 = 70^\circ \\
T = 70.21 \text{ m} \\
L_{cc} = 125 \text{ m} \\
\text{Curve } #1 \ D = 8^\circ \ldots \ d = 4^\circ \\
d_1 = (18.39)(8)(40) = 3^\circ 40'40'' \\
d_2 = (11.61)(8)(40) = 2^\circ 19'20'' \\
33 + 6.61 \ P.T. \quad 35^000'00'' = \Delta_T/2 \ldots \text{curve } #2 \ D = 12^\circ \ldots \ d = 6^\circ \\
33 + 00 \quad 33^040'40'' \quad d_1 = (8.39)(12)/20 = 5^002'00'' \\
32 + 80 \quad 29^040'40'' \quad d_2 = (6.61)(12)/20 = 3^058'00'' \\
32 + 60 \quad 25^040'40'' \\
32 + 56.61 \ P.C.C.2 \quad 25^000'00'' = (\Delta_1+\Delta_2)/2 \ldots \text{curve } #3 \ D = 8^\circ \ldots \ d = 4^\circ \\
32 + 50 \quad 21^002'00'' \quad d_1 = (3.39)(8)/40= 0^040'40'' \\
32 + 40 \quad 15^002'00'' \quad d_2 = (6.61)(8)/40= 1^019'20'' \\
32 + 31.61 \ P.C.C.1 \quad 10^000'00'' = \Delta_1/2 \ldots \\
32 + 20 \quad 7^040'40'' \\
32 + 00 \quad 3^040'40'' \\
31 + 81.61 \ P.C. \quad 0^000'00''

### NOTE: The checks of P.C.C.1, P.C.C.2, and P.T. are essential!
Field procedure for compound curves is essentially the same as for simple curves. The transit has to be set up at P.C.C.1, and P.C.C.2 in order to run in the curves.

PROBLEMS

1. Determine the radius of the central curve of a symmetrical compound curve which passes through point "A", and determine the stationing of point "A".

![Diagram of a compound curve with labeled measurements and angles.]