4.6 Transformation Between Geographic and UTM Coordinates

4.6.1 Conversion from Geographic to UTM Coordinates

- Used for converting $\phi$ and $\lambda$ on an ellipsoid of known $f$ and $a$, to UTM coordinates. Negative values are used for western longitudes.

- These equations are accurate to about a centimeter at 7° of longitude from the central meridian

Where $\phi_o = 0$ (latitude of the central meridian at the origin of the $x$, $y$ coordinates)

$M = $ True distance along central meridian from the equator to $\phi$ (across from the point)

$M_o = 0$ (M at $\phi_o$)

$\lambda_o = $ longitude of central meridian (for UTM zone)

$k_o = 0.9996$ (scale factor at the central meridian)
4.6 Transformation Between Geographic and UTM Coordinates

FROM EQUATION SHEET

Radius of Curvature in the plane of the meridian

\[ R_m = a \frac{1 - e^2}{(1 - e^2 \sin^2 \phi)^{\frac{3}{2}}} \]

Radius of curvature on the plane of the prime vertical

\[ N = R_N = \frac{a}{\sqrt{1 - e^2 \sin^2 (\phi)}} \]

Radius of curvature at a given azimuth

\[ R_\alpha = \frac{R_m R_N}{R_m \sin^2 (\alpha) + R_N \cos^2 (\alpha)} \]
4.6 Transformation Between Geographic and UTM Coordinates

4.6.1 Conversion from Geographic to UTM Coordinates

\[ T = \tan^2 \phi \]
\[ C = e^{i2} \cos^2 \phi \]
\[ A = (\lambda - \lambda_0) \cos \phi ........... \text{where } \lambda \text{ and } \lambda_0 \text{ are in radians} \]

\[ M = a \left[ \left( 1 - \frac{e^2}{4} - \frac{e^4}{64} - \frac{e^6}{256} - \ldots \right) \phi - \left( \frac{3e^2}{8} + \frac{3e^4}{32} + \frac{45e^6}{1024} + \ldots \right) \sin 2\phi + \right. \]
\[ \left. \left( \frac{15e^4}{256} + \frac{45e^6}{1024} + \ldots \right) \sin 4\phi - \left( \frac{35e^6}{3072} + \ldots \right) \sin 6\phi + \ldots \right] \]

where \( \phi \) is in radians
4.6 Transformation Between Geographic and UTM Coordinates

4.6.1 Conversion from Geographic to UTM Coordinates

**Northing and Easting**

\[
x = k_o R_N \left[ A + \left( 1 - T + C \right) \frac{A^3}{6} + \left( 5 - 18T + T^2 + 72C - 58e^2 \right) \frac{A^5}{120} \right]
\]

\[
y = k_o \left\{ M - M_o + R_N \tan \phi \left[ \frac{A^2}{2} + \left( 5 - T + 9C + 4C^2 \right) \frac{A^4}{24} + \left( 61 - 58T + T^2 + 600C - 330e^2 \right) \frac{A^6}{720} \right] \right\}
\]

**UTM Scale Factor**

\[
k = k_o \left[ 1 + \left( 1 + C \right) \frac{A^2}{2} + \left( 5 - 4T + 42C + 13C^2 - 28e^2 \right) \frac{A^4}{24} + \left( 61 - 148T + 16T^2 \right) \frac{A^6}{720} \right]
\]

**Or in terms of Latitude and Longitude**

\[
k = k_o \left[ 1 + \left( 1 + e^2 \cos^2 (\phi) \right) \frac{x^2}{2k_o^2 R_N^2} \right]
\]
4.6 Transformation Between Geographic and UTM Coordinates

4.6.2 Conversion from UTM to Geographic Coordinates

Used for converting UTM coordinates on an ellipsoid of known f and a, to \( \phi \) and \( \lambda \). Negative values are used for western longitudes.

These equations are not as accurate as the geographic to UTM conversion

Where \( \phi_1 \) = footprint latitude which is the latitude at the central meridian which has the same y coordinate of the point

\( \mu \) = the rectifying latitude
4.6 Transformation Between Geographic and UTM Coordinates

4.6.2 Conversion from UTM to Geographic Coordinates

\[ M = M_o + \frac{y}{k_o} \]

\[ \mu = \frac{M}{a \left(1 - \frac{e^2}{4} - \frac{3 e^4}{64} - \frac{5 e^6}{256} - \ldots\right)} \]

\[ e_1 = \frac{1 - \sqrt{1 - e^2}}{1 + \sqrt{1 - e^2}} \]

\[ \phi_1 = \mu + \left(3 \frac{e_1}{2} - 27 \frac{e_1^3}{32} + \ldots\right) \sin(2\mu) + \left(21 \frac{e_1^2}{16} - 55 \frac{e_1^4}{32} + \ldots\right) \sin(4\mu) \]

\[ + \left(151 \frac{e_1^3}{96} + \ldots\right) \sin(6\mu) + \left(1097 \frac{e_1^4}{512} - \ldots\right) \sin(8\mu) + \ldots \]

where \( \mu \) is in radians

\( C_1, T_1, R_{N_1}, \) and \( R_1 \) are \( C, T, R_N, \) and \( R_m \) calculated at the footprint latitude \( (\phi_1) \)

\[ D = \frac{x}{R_{N_1} k_o} \]
4.6 Transformation Between Geographic and UTM Coordinates

4.6.2 Conversion from UTM to Geographic Coordinates

\[
\phi = \phi_1 - \left( R_N \frac{\tan(\phi_1)}{R_1} \right) \left[ \frac{D^2}{2} - \left( 5 + 3T_1 + 10C_1 - 4C_1^2 - 9\epsilon^2 \right) \frac{D^4}{24} \right. \\
+ \left. \left( 61 + 90T_1 + 298C_1 + 45T_1^2 - 252\epsilon^2 - 3C_1^2 \right) \frac{D^6}{720} \right]
\]

\[
\lambda = \lambda_0 + \frac{D - (1 + 2T_1 + C_1)D^3}{6 + \left( 5 - 2C_1 + 28T_1 - 3C_1^2 + 8\epsilon^2 + 24T_1^2 \right) \frac{D^5}{120}} \cos(\phi_1)
\]
4.6 Transformation Between Geographic and UTM Coordinates

4.6.3 UTM Map Scale Factor

The elevation factor can be approximated using the average radius of the earth \((R=6,367,272\text{m})\) and elevation above the geoid rather than the elevation above the ellipsoid. This is done because of the relatively small value of \(N\) in comparison to \(H\), and because the geoid height is usually used for elevation.

![Image of a map with a compass rose and text overlay](image)

\[
\text{Elevation factor} = \frac{R_\alpha}{R_\alpha + h} \quad \text{Approx. Elevation factor} = \frac{R_E}{R_E + H}
\]

(The UTM scale factor can also be approximated using the average radius of the earth.)

Approximate UTM Scale factor \(k = k_0\left\{1 + \frac{x^2}{2R^2}\right\}\)

The grid scale factor for UTM maps can then be computed using the approximate or true values

\(Grid\ Factor = (\text{scale factor}) \times (\text{Elevation factor})\)

\[
\text{Ground Distance} = \frac{\text{Grid distance}}{\text{Grid factor}}
\]
4.6 Transformation Between Geographic and UTM Coordinates

4.6.3 UTM Map Scale Factor

[Review]

Central Meridian

Ground surface

Mean sea level

Projection surface

Ellipsoid surface

$R$

$h$

$H$

$N$

$k_0 = 0.9996$

$k_0 = 1.00$

$k_0 = 1.00$
4.6 Transformation Between Geographic and UTM Coordinates

4.6.4 EXAMPLE A

**GIVEN:**
Points on map from geodetic bench marks
Map: NAD27, 1:250,000 NTS map of 72H (Willow Bunch Lake)
φ = 49°15’N       λ = 104°20’W
Approx. Elevation h = 2430 ft = 740.66m

**FIND:**

a) **UTM coordinates for point A, where:**

a = 6,378,206.4 m       1/f = 294.9786982
φ = 49°15’N           = 49.25°        = 0.859575 radians
λ = 104°20’W       = -104.3333°    = -1.82096 radians (UTM zone 13)
λₒ = 105° W          kₒ= 0.9996      φₒ = 0°
4.6 Transformation Between Geographic and UTM Coordinates

4.6.4 EXAMPLE A
4.6 Transformation Between Geographic and UTM Coordinates

4.6.4 EXAMPLE A

\[ e^2 = 2f - f^2 = 0.00676865 \]
\[ e'^2 = \frac{e^2}{1 - e^2} = 0.00681478 \]
\[ R_M = a \frac{1 - e^2}{\left(1 - e^2 \sin^2 \phi \right)^{\frac{3}{2}}} = 6,372,127.842 \text{ m} \]
\[ R_N = N = a \frac{1}{\sqrt{1 - e^2 \sin^2 \phi}} = 6,390,630.874 \text{ m} \]
\[ T = \tan^2 \phi = 1.34689285 \]
\[ C = e^2 \cos^2 \phi = 0.00290374 \]
\[ A = \left(\lambda - \lambda_o\right) \cos \phi = 0.0075952 \]
\[ M = a \left[ \left(1 - e^2 \right) - 3 \frac{e^4}{64} - 5 \frac{e^6}{256} - \cdots \right] \phi - \left(3 \frac{e^2}{8} + 3 \frac{e^4}{32} + 45 \frac{e^6}{1024} + \cdots \right) \sin 2\phi + \right] \]
\[ M_o = 0 \]

used in this equation is not to be confused with geoidal height.
4.6 Transformation Between Geographic and UTM Coordinates

4.6.4 EXAMPLE A

\[ x = k_o N \left( A + (1 - T + C) \frac{A^3}{6} + \left( 5 - 18T + T^2 + 72C - 58\epsilon^2 \right) \frac{A^5}{120} \right) \]

= 48,518.5439 m

add a false easting of 500,000m

**E = 548,518.544 m**

\[ y = k_o \left[ (M - M_o) + N \tan \phi \left( \frac{A^2}{2} + \left( 5 - 9C + 4C^2 \right) \frac{A^4}{24} + \left( 61 - 58T + T^2 + 600C - 330\epsilon^2 \right) \frac{A^6}{720} \right) \right] \]

**N = 5,455,242.563 m**
4.7 Application of UTM Coordinates

http://www.geod.nrcan.gc.ca/apps/gsrug/geo_e.php

GSRUG - Geodetic Survey Routine: UTM and Geographic

This program will compute the conversion between Geographic coordinates, latitude and longitude and Transverse Mercator Grid coordinates.

The user may choose this standard projection or may choose a 3 degree as defined for Canada. The parameters of scale, central meridian, false easting and false northing may define any TM projection and are already defined within the program for two standard projections, UTM and 3 degree.

Geographic to UTM computation output

**Input Geographic Coordinates**

LATITUDE: 49 degrees 15 minutes 0 seconds NORTH
LONGITUDE: 104 degrees 20 minutes 0 seconds WEST
ELLIPSOID: CLARKE 1866
ZONE WIDTH: 6 Degree UTM

**GSRUG UTM coordinates:**

<table>
<thead>
<tr>
<th>UTM Zone: 13</th>
<th>Easting: 548518.573 meters EAST</th>
<th>Northing: 5455242.533 meters NORTH</th>
</tr>
</thead>
</table>

**Output- Calculated UTM coordinates:**

<table>
<thead>
<tr>
<th>UTM Zone: 13</th>
<th>Easting: 548518.544 meters EAST</th>
<th>Northing: 5455242.563 meters NORTH</th>
</tr>
</thead>
</table>
4.7 Application of UTM Coordinates

4.7.1 EXAMPLE B

FIND:

b) Latitude, longitude and height of point A with respect to NAD 83 ellipsoid

\[ a' = 6378137 \text{m} \quad 1/f' = 298.257 \]

GIVEN

\[ dx = 4 \text{m} \quad dy = 159 \text{m} \quad dz = 188 \text{m for Saskatchewan} \]

Note: \( dx = \delta x \)
4.7 Application of UTM Coordinates

\[ \delta a = a' - a = 6378137 - 6378206.4 = -69.4 \text{ m} \]

\[ \delta f = f' - f = \frac{1}{298.257} - \frac{1}{294.979} = -3.72587 \times 10^{-5} \]

\[ R_N = \frac{a}{\sqrt{1 - e^2 \sin^2(\phi)}} = 6390630.874 \text{ m} \]

\[ R_M = a \frac{1 - e^2}{\left(1 - e^2 \sin^2 \phi \right)^{\frac{3}{2}}} = 6372127.842 \text{ m} \]

\[ \delta \phi = \frac{\left( (- \delta x \sin \phi \cos \lambda - \delta y \sin \phi \sin \lambda) + \delta z \cos \phi \right)}{R_N} + \delta a \frac{R_N e^2 \sin \phi \cos \phi}{a} + \delta f \left( \frac{R_M}{1 - f} + R_N (1 - f) \right) \sin \phi \cos \phi \]

\[ \delta \phi = 7.5149 \times 10^{-7} \text{ rad} = 4.306 \times 10^{-5} \text{ deg.} = 0.155'' \]

\[ \delta \lambda = \frac{-\delta x \sin \lambda + \delta y \cos \lambda}{(R_N + h) \cos \phi} \]

\[ \delta \lambda = -8.5059 \times 10^{-6} \text{ rad} = -0.0004874^\circ = 1.75'' \]

\[ \delta h = \delta x \cos \phi \cos \lambda + \delta y \cos \phi \sin \lambda + \delta z \sin \phi - \delta a \frac{a}{R_N} + \delta f \left( 1 - f \right) R_N \sin^2 \phi \text{ m} \]

\[ \delta h = -25.704 \text{ m} \]

\[ \phi' = \phi + \delta \phi = 49^\circ 15' + 0.155'' = 49^\circ 15' 0.16'' \text{ N} \]

\[ \lambda' = \lambda + \delta \lambda = -104^\circ 20' + (-1.75'') = 104^\circ 20' 1.75 \text{ W} \]

\[ h' = h + \delta h = 740.66m + (-25.704m) = 714.956 \text{ m} \]
4.7 Application of UTM Coordinates

4.7.1 EXAMPLE B con’t.

National Transformation: NAD27 - NAD83 (NTv2),
NTv2
Computation output
Input Coordinates
LATITUDE: 49 degrees 15 minutes 00.000000 seconds NORTH
LONGITUDE: 104 degrees 20 minutes 00.000000 seconds WEST
Transformation: NAD27 -> NAD83

NAD 83 Output data:
LATITUDE: 49° 15’ 0.11403“ N
Shift: 0.11403 seconds
Standard deviation: 0.078m
LONGITUDE: 104° 20’ 1.87927” W
Shift: 1.87927 seconds
Standard Deviation: 0.208m
4.7 Application of UTM Coordinates

4.7.1 EXAMPLE B con’t.

National Geodetic Survey
http://www.ngs.noaa.gov/cgi-bin/nadcon.prl

Works up to 50° N
In Western Canada
4.7 Application of UTM Coordinates

4.7.2 EXAMPLE  C

FIND:

c) Latitude and longitude of point B with respect to NAD 27

E = 560,000m  N = 5,470,000m

a = 6,378,206.4  1/f = 294.979

λ₀ = 105°W  k₀ = 0.9996  M₀ = 0°
4.7 Application of UTM Coordinates

4.7.2 EXAMPLE C

\[ M = M_o + \frac{y}{k_o} = 5472188.876 \]
\[ e^2 = 2f - f^2 = 0.00676865 \]
\[ e'^2 = \frac{e^2}{1 - e^2} = 0.00681478 \]
\[ \mu = \frac{M}{a \left(1 - \frac{e^2}{4} - \frac{3e^4}{64} - \frac{5e^6}{256} - \cdots \right)} \]
\[ e_i = \frac{1 - \sqrt{1 - e^2}}{1 + \sqrt{1 - e^2}} = \]
\[ \phi_i = \mu + \left(3 \frac{e_i}{2} - 27 \frac{e_i^3}{32} + \cdots \right) \sin(2\mu) + \left(21 \frac{e_i^2}{16} - 55 \frac{e_i^4}{32} + \cdots \right) \sin(4\mu) \]
\[ + \left(151 \frac{e_i^3}{96} + \cdots \right) \sin(6\mu) + \left(1097 \frac{e_i^4}{512} - \cdots \right) \sin(8\mu) + \cdots = \]
\[ R_i = a \frac{1 - e^2}{\left(1 - e^2 \sin^2 \phi_i \right)^{\frac{3}{2}}} = \]
4.7 Application of UTM Coordinates

4.7.2 EXAMPLE C

\[ T_1 = \tan^2 \phi_1 = 1.35976 \]
\[ C_1 = e^{i^2} \cos^2 \phi_1 = 0.0028879 \]
\[ x = X - \text{(false easting)} = 60,000 \]
\[ D = \frac{x}{N_1 k_0} = 0.0093924 \]

\[
\phi = \phi_1 - \left( N_1 \frac{\tan(\phi_1)}{R_1} \right) \begin{bmatrix}
\frac{D^2}{2} - \left( 5 + 3T_1 + 10C_1 - 4C_1^2 - 9e^{i^2} \right) \frac{D^4}{24} \\
+ \left( 61 + 90T_1 + 298C_1 + 45T_1^2 - 252e^{i^2} - 3C_1^2 \right) \frac{D^6}{720} 
\end{bmatrix}
\]

\[ \phi = 0.8618735 \text{ rad } = 49.381713^\circ = 49^\circ 22' 54.168'' \text{ N} \]

\[ D = (1 + 2T_1 + C_1) \frac{D^3}{6} + \left( 5 - 2C_1 + 28T_1 - 3C_1^2 + 8e^{i^2} + 24T_1^2 \right) \frac{D^5}{120} \]

\[ \lambda = \lambda_o + \frac{D^5}{120} \cos(\phi_1) \]
\[ \lambda = -1.832596 \text{ rad } + 0.0144274 = -1.818168 \text{ rad } = -104.17337^\circ = 104^\circ 10'24.134'' \text{ W} \]
4.7 Application of UTM Coordinates

4.7.2 EXAMPLE  C

GSRUG - Geodetic Survey Routine: UTM and Geographic
UTM to Geographic computation output
Input Geographic Coordinates
UTM Zone: 13
Northing: 5470000 meters
Easting: 560000 meters
ELLIPSOID: CLARKE 1866
ZONE WIDTH: 6 Degree UTM

Output geographic coordinates:
LATITUDE: 49° 22’ 54.168061” N
LONGITUDE: 104° 10’ 24.134352” W

Calculated geographic coordinates:
LATITUDE:
LONGITUDE:
Given:
A-B has a calculated grid Azimuth $\alpha = 37^\circ 52' 59.5''$

Find:
“True” Azimuth of line from A to B” (seconds)

$$\Delta \alpha = \theta = \Delta \lambda \sin \varphi_m \sec \frac{\Delta \varphi}{2} + (\Delta \lambda)^3 \cdot F$$

$$\Delta \alpha = \theta = 2688'' \sin 49.3158333 \cdot 1 + (\Delta \lambda)^3 \cdot F$$

$$F = \frac{1}{12} \sin \varphi_m \cos^2 \varphi_m \sin^2 1''$$

$$\theta = 2038 '' = 0^\circ 33' 58''$$
4.8 Map Azimuth and Scale Factors of Line A

MAP AZIMUTH AND SCALE FACTORS OF LINE A - B

Corrected (Astronomic) Azimuth A to B
\[ \alpha = 37^\circ 52' 59.5 + 0^\circ 33' 58" = \]

Precise scale Factor (S.F.)

UTM Scale factor

\[ k = k_0 \left[ 1 + \left( 1 + C \right) \frac{A^2}{2} + \left( 5 - 4T + 42C + 13C^2 - 28e^2 \right) \frac{A^4}{24} + \left( 61 - 148T + 16T^2 \right) \frac{A^6}{720} \right] \]

Precise Elevation Factor (E.F.)

\[ R_\alpha = \frac{R_M}{R_M \sin^2(\alpha) + R_N \cos^2(\alpha)} = R_\alpha = \]

From Geodetics Canada GPS - H v.2 software:

H = 740.66m  h = 722.71m  N = -17.95m

Precise Elevation Factor (E.F.) = \[ \frac{R_\alpha}{R_\alpha + h} = \]

True grid factor = S.F. \times E.F.

0.99963 \times 0.999887 =
4.8 Map Azimuth and Scale Factors of Line A

MAP AZIMUTH AND SCALE FACTORS OF LINE A - B

Approximate scale factors based on a spherical earth

UTM Scale Factor (S.F.)

\[ M_p = k_o \left(1 + \frac{x^2}{2R^2}\right) = 0.9996 \left(1 + \frac{48,548.5^2}{2 \times 6,367,272^2}\right) = \]

Elevation Factor (E.F.)

\[ \frac{6,367,272}{6,367,272 + 740} = \]

Approximate grid factor

0.99963 \times 0.999884 =

Rough Conversion

\[ \theta = 52.13d \tan \phi = 52.13 \times \frac{(48518.5) \times 3.2808}{5280} \tan(49^\circ15') \]

\[ \theta = 52.13 \times 30.15 \times 1.1606 = 1824.1'' = \]
More Precise Spherical Conversion

\[ \theta'' = \Delta \lambda'' \sin \left( \frac{\varphi_A + \varphi_B}{2} \right) \]

\[ \theta'' = 2688'' \sin \left( \frac{49.25 + 49.38166667}{2} \right) = 2038 \]

Corrected (Astronomic) Azimuth A to B

\[ \alpha = 37^\circ 52' 59.5 + 0^\circ 33' 58'' = 38^\circ 26' 57.5'' \]