EE 212

PASSIVE AC CIRCUITS

Condensed Text

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About the EE 212 Condensed Text

The major topics in the course EE 212 include circuit theory, power circuit basics, single and three phase power, transformer, and per unit system. Publishers have not been able to provide a text book that contains all of these topics. The book titled "***Introduction to Electric Circuits***" **by *R.C. Dorf and J.A. Svoboda*** contains many of the topics listed in this course, and has therefore been used as the recommended text for EE 212. This book, however, does not cover all the materials required for EE 212, and many previous students have found the text not very suitable for the course. This Condensed Text has, therefore, been prepared to provide the necessary class materials in a condensed form. It is very important for the students to attend all the lectures in order to understand and to discuss on the contents in this Condensed Text. Students are also advised to refer to other texts that contain the materials included in the course.

Class Website

All class assignments, solutions, and notices will be posted on the following class website:

www.engr.usask.ca/classes/ee/212

Please check the above website regularly to be sure you don't miss anything. The newest messages will be placed on the top. It is the responsibility of the student to keep up-to-date on the material contained in this website.

MAJOR TOPICS

1. Introduction, AC waveforms, phase shift in R, L, C circuits
2. Steady-state circuit analysis - time domain, phasor concept
3. Phasor diagrams, impedance/admittance, resistance and reactance in complex plane
4. Power factor, real and reactive power
5. Loop and Nodal analysis
6. Thevenin's/Norton's theorem, maximum power transfer theorem, wye-delta transformation
7. Superposition theorem, multiple sources with different frequency, non-sinusoidal sources
8. Series and parallel resonance
9. Coupled circuits
10. Transformer action, equivalent circuit, Losses
11. Transformer open and short circuit tests, efficiency and voltage regulation
12. 3-phase systems, Y-delta connections/transformations
13. Multiple 3-phase loads
14. Power Measurement, Wattmeter connections in 1-phase and 3-phase balanced/unbalanced systems
15. Per Unit system

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Introduction

Linear and Non-Linear Circuits

A linear circuit has the following properties:

- obeys Ohm's Law
  i.e. voltage (v) α current (i)
  or $v = Ri$, where R is the circuit resistant

- if the current or voltage in any part of the circuit is sinusoidal, the current and voltage in every other part of the circuit is sinusoidal, and of the same frequency

Non-linear circuits do not obey Ohm's Law.
e.g. circuit consisting of p-n diode, transistor, etc.

Types of Circuit Elements

Electric circuits can be categorized as active and passive.

Active circuit elements supply energy to the circuit. These elements can either be voltage sources or equivalent current sources. Some examples of active circuit elements are battery, transistor, IC components.

Passive circuit elements absorb energy. Resistors, inductors, and capacitors are examples of passive circuit elements. ***Passive AC circuits*** *are composed of resistors, inductors and capacitors, and are energized by alternating current sources*.

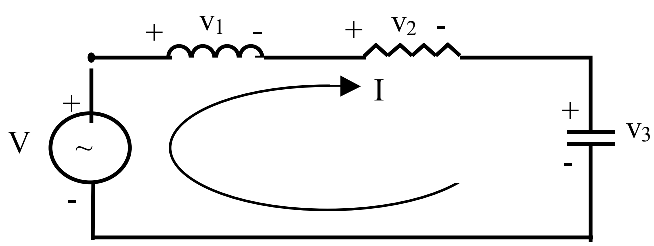
"Steady-state analysis of linear AC circuits" is within the scope of this course.

Kirchoff's Laws

Kirchoff's two important circuit theory laws are known as the Kirchoff's Voltage Law (KVL) and the Kirchoff's Current Law (KCL).

1. Kirchoff's Voltage Law (KVL)

The sum of the instantaneous voltages around any closed loop is zero.



Applying KVL to the above circuit, the following equation is obtained.

$$- V + v_1 + v_2 + v_3 = 0$$

The following sign conventional is used in this class when applying KVL to a circuit:

Sign convention:

For a current going from +ve to –ve, voltage is +ve

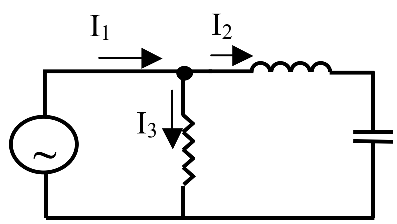
For a current going from -ve to +ve, voltage is –ve

In a voltage source, the polarities are known. The current may go from +ve to –ve or vice-versa. In a passive element (R, L or C), the current always goes from +ve to –ve. This is because a current always flows from a higher potential to a lower potential in a circuit. A +ve sign indicates the higher potential node.

This law can be used to calculate the current in a loop from which the individual currents in each element can be calculated.

2. Kirchoff's Current Law (KCL)

The sum of the instantaneous currents at any node is zero.



Applying KCL to the above circuit, the following equation is obtained.

$$- I_1 + I_2 + I_3 = 0$$

The following sign conventional is used in this class when applying KCL to a circuit:

Sign convention:

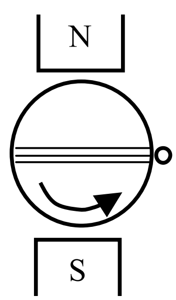
current exiting a node is taken as +ve

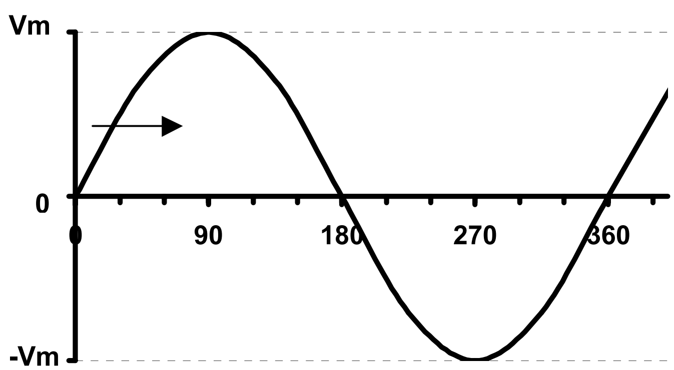
current entering a node is taken as -ve

This law can be used to calculate the voltage at the different nodes in a circuit.

AC Waveforms

The principle of AC voltage generation from an AC source is shown in the figure below. The figure on the left represents an alternator. When a conductor coil wound around the rotating armature moves across the magnetic field, a voltage is induced on the conductor. At the conductor position shown in the figure, the voltage induced is zero, since the conductor does not cut through the magnetic field. As the armature rotates anti-clockwise, the conductor will increasingly cut through stronger magnetic field, and consequently, the voltage induced will increase continuously. At 90^0 rotation of the armature, the conductor will lie just below the North Pole, and the voltage induced is maximum. This is known as the peak value V_m of the voltage waveform. Further rotation takes the conductor away from the magnetic field, and therefore, the voltage induced will decrease. At 180^0 rotation, the conductor does not cut through the magnetic field, and the voltage induced is again zero. After 270^0, the conductor is just below the opposite pole (South Pole), and therefore, the voltage induced is maximum in the reverse direction. After 360^0, the armature position becomes the same as that of 0^0 (starting position). This cycle keeps repeating. The generated voltage waveform is therefore a periodic sinusoidal waveform as shown in the plot below.





A sinusoidal voltage waveform is mathematically expressed as:

$$v(t) = V_m \sin \omega t$$

where, angular frequency, $\omega = 2\pi f = 2\pi/T$ rad/sec

and, T is the time period of the waveform.

Standard Frequency f = 60 Hz in North & South America

= 50 Hz in Europe, Asia, Africa, Australia

The ***instantaneous value*** of a sinusoidal waveform changes continuously with time. For example, the value of an AC voltage can be anywhere between zero and the peak value at any instant in time. An AC voltage is normally expressed by one value. For example, the voltage supply in a residential home power outlet is 120 volts.

A set of varying data (or a distribution) is often expressed by its average value. However, the average value of a sinusoidal waveform is zero, and therefore, cannot be used to represent an AC waveform.

A sinusoidal waveform is expressed by an effective value, which is known as the ***root mean square***, (RMS) value. An AC RMS current dissipates the same amount of heat in a resistor as a DC current. The RMS value is therefore equivalent to a DC value in terms of power dissipation.

$$\text{RMS value} = \frac{\text{Peak Value}}{\sqrt{2}}$$

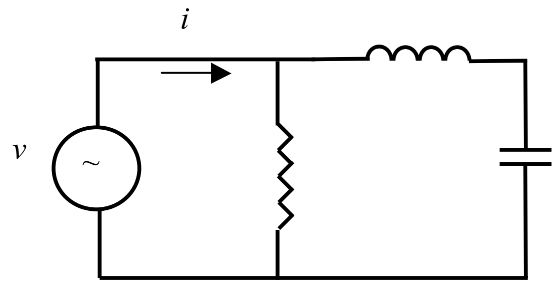
i.e. $$V_{rms} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{rms} = \frac{I_m}{\sqrt{2}}$$

When a sinusoidal input is applied to a linear circuit, the response is also sinusoidal. The response has the same frequency as the input signal, but may have different magnitude & phase angle.

The input signal in the circuit below is a sinusoidal voltage expressed as:

$$v = V_m \sin \omega t$$

where, V_m is the peak value of the voltage waveform, and ω is the angular frequency.



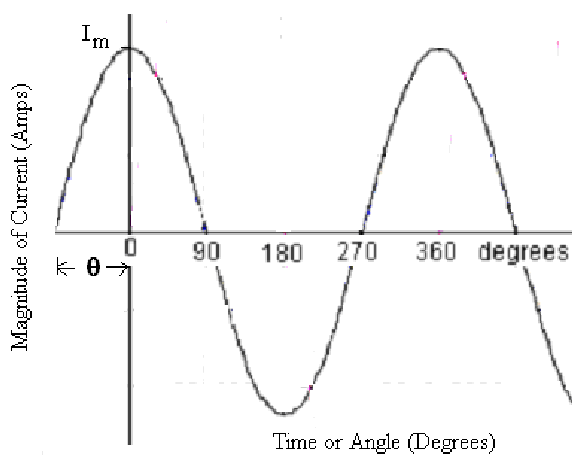
The response *i* of the circuit is also sinusoidal, and of the same angular frequency.

$$i = I_m \sin(\omega t + \theta)$$

where, I_m is the peak value of the current waveform. θ is the phase angle between *v* and *i,* and is also known as the ***power factor angle***.

The general form of a sinusoidal equation is

$$i = I_m \sin(\omega t + \theta)$$



The effect of the phase angle *θ* is to slide the waveform sideways. The main parameters of a sinusoid are the magnitude and the phase.

Power Factor

Power factor is the cosine of the angle between the current and the voltage.

i.e. power factor, p.f. = cos θ

where θ is known as the power factor angle.

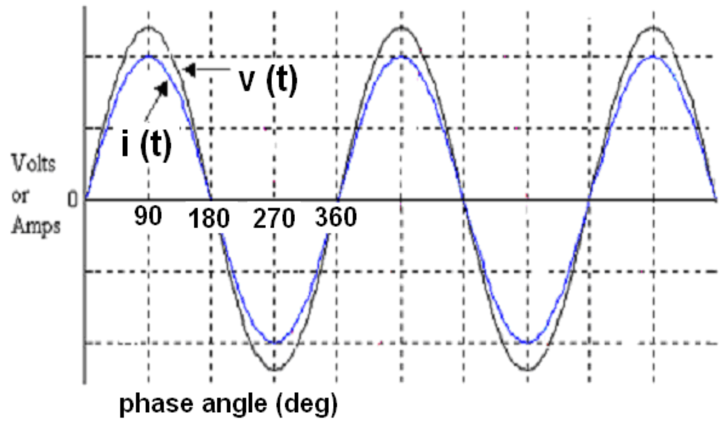
If θ is + ve, *i* leads *v*, and the power factor is leading.
e.g. in a *capacitive circuit*
(the current through a capacitor leads the voltage across it by 90^0)

If θ is - ve, *i* lags *v,* and the power factor is lagging.
e.g. in an *inductive circuit*
(the current through an inductor lags the voltage across it by 90^0)

If θ is zero, *i* is in phase with *v,* and the power factor is 1 or unity ($\cos 0^0 = 1$).
e.g. in a *resistive circuit*
(the current through a resistor is in phase with the voltage across it)

Phase Shift in R, L, C Circuits

Phase Shift in a Resistive Circuit



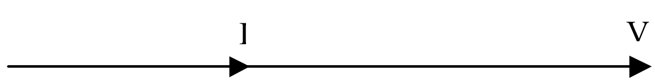
The voltage waveform, $v(t) = V_m \sin \omega t$

The current waveform, $i(t) = I_m \sin \omega t$

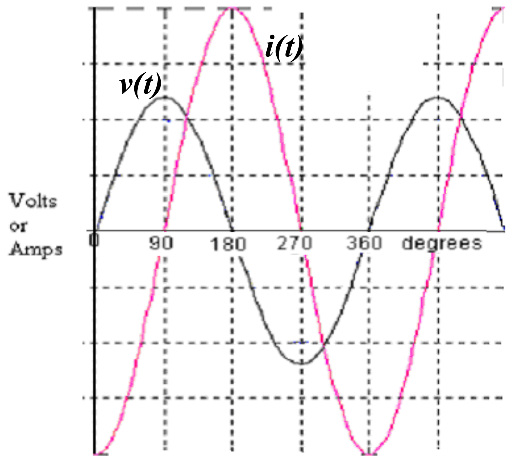
The voltage and current are in phase.

Power factor, p.f. = $\cos \theta = \cos 0^0 = 1$ i.e. unity power factor

The phasor diagram for the voltage and current in a resistive circuit is shown below.



Phase Shift in an Inductive Circuit



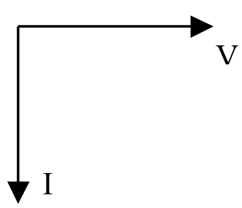
The voltage waveform, $v(t) = V_m \sin \omega t$

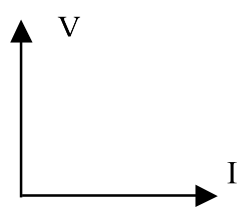
The current waveform, $i(t) = I_m \sin (\omega t - 90^0)$

The current lags the voltage by 90 degrees.

Power factor, p.f. = $\cos \theta = \cos 90^0 = 0$, i.e. zero power factor

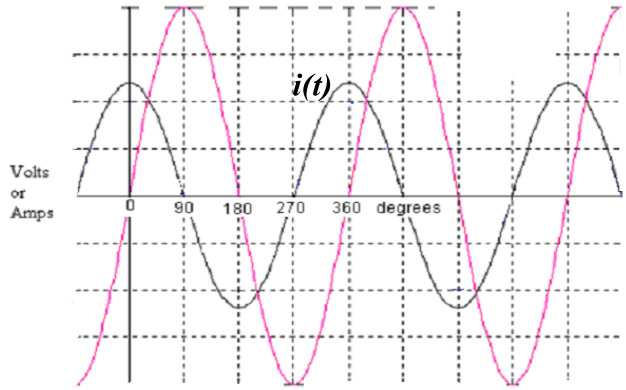
The phasor diagram for the voltage and current in an inductive circuit is shown below.





Phase Shift in a Capacitive Circuit

v(t)



$v(t) = V_m \sin \omega t$

$i(t) = I_m \sin (\omega t + 90^0)$

The current leads the voltage by 90 degrees

Power factor, p.f. = $\cos \theta = \cos 90^0 = 0$,   i.e. zero power factor

The phasor diagram for the voltage and current in a capacitive circuit is shown below.

Steady-state Circuit Analysis

When a circuit has more than one element, a circuit analysis is required to determine circuit parameters (voltage, current, power, etc.) in different parts of the circuit. There are different methods for circuit analysis. The Time Domain Method and the Phasor Method are discussed here.

Time Domain Method:

- applicable to both transient and steady-state circuit analysis
- very useful for transient analysis
- difficult method (requires differentiation & integration of sinusoidal functions)

Phasor Method:

- applicable to steady-state circuit analysis
- easy method
    - sinusoidal functions represented by phasors
    - differentiation & integration replaced by multiplication and division

The Phasor Method is very useful for steady-state analysis of linear AC circuits.

The Phasor Concept

A sinusoidal waveform can be represented by its magnitude (usually the RMS value) and the phase angle.

Sinusoidal voltage and current in the Time Domain,

$$v = V_m \sin \omega t$$

$$i = I_m \sin (\omega t + \theta)$$

In Phasor (or Frequency Domain),

$$V = \frac{V_m}{\sqrt{2}} \underline{/0^0}$$

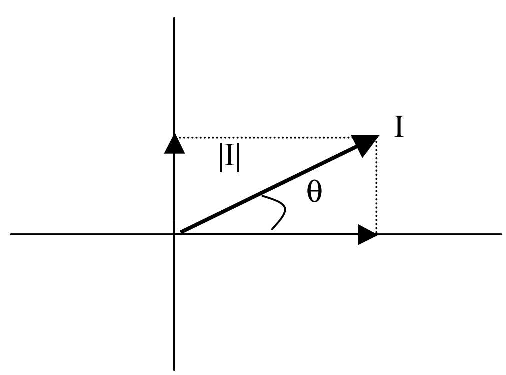
$$I = \frac{I_m}{\sqrt{2}} \underline{/\theta^0}$$

A phasor (voltage or current) can be expressed either in the polar form (magnitude and angle) or in the rectangular form (real and imaginary parts).

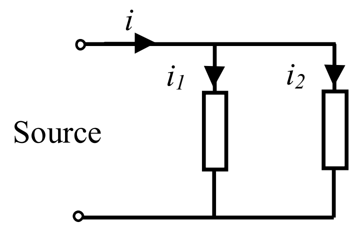
e.g. A current phasor in the *polar form* is $\mathbf{I = |I| \ \underline{/\theta^0}}$

and in the *rectangular form* is $\mathbf{I = |I| \cos \theta + j \ |I| \sin \theta}$

and the *Phasor Diagram* is shown below:



Problem.



Calculate the total current *i* in the above circuit and plot its waveform, if

i_1 = 50 sin (377 t + 20^0) A and

i_2 = 10 sun (377 t + 10^0) A.

Circuit Analysis Using the Phasor Method

1. A linear AC circuit may contain a number of passive elements (resistor R, Inductor L, and/or capacitor C) connected in a network.

2. The input(s) to the circuit are sinusoidal voltage or current waveform expressed as v(t) and i(t) in the time domain.

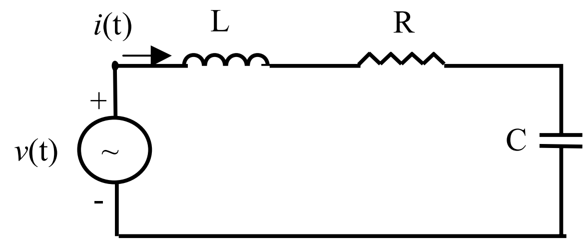
3. The circuit is converted from the time-domain to a phasor representation. This is done by the following transformations from time-domain to phasor:

Time-domain	**Phasor**
Time dependent voltage v(t) and current i(t)	Phasor voltage V and current I
Inductance L in Henry	Inductive reactance jωL in ohms
Capacitance C in Farad	Capacitive reactance 1/(jωC) in ohms

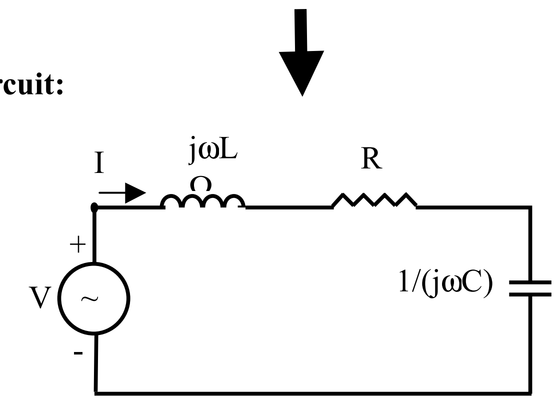
4. All the calculations are done in the phasor form.

5. Final result is converted back to the time domain, if required.

Circuit in the time domain:



Phasor representation of the circuit:



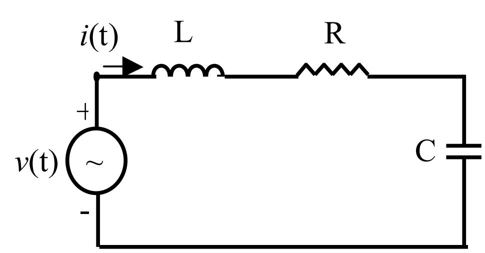
Problem.

Calculate the instantaneous current in the circuit below when the instantaneous value of the source voltage is 50 volts, where

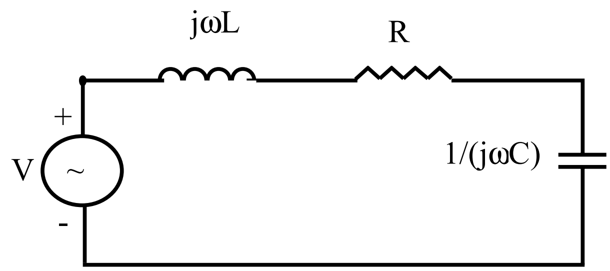
$v(t) = 100$ Sin $(377t + 30)$

R = 10, L = 1/37.7 h,

and C = 1/7540 f.



Impedance and Admittance



The impedance of the above circuit is expressed as:

$$Z = R + j\omega L - j/(\omega C)$$

$$= R + jX_L - jX_C$$

$$= R + jX$$

where, X is known as the reactance, X_L the inductive reactance, and X_C the capacitive reactance.

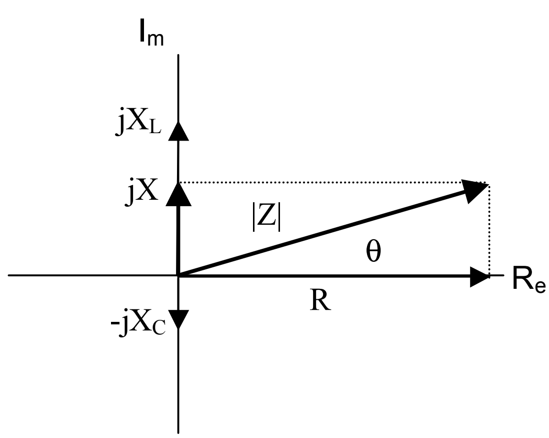
Z is a complex number (shown in the complex plane below). It can be expressed in

the rectangular form, $Z = R + jX$

or the polar form, $Z = |Z| \underline{/\theta^0}$,

where θ is the power factor angle.

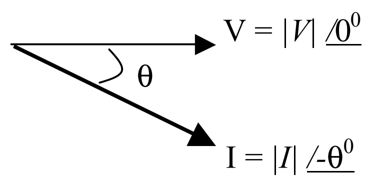
The following figure shows the impedance Z in the complex plane.



The polarity of the power factor angle θ has no effect on the magnitude of the power factor, since

$$\cos(\theta) = \cos(-\theta).$$

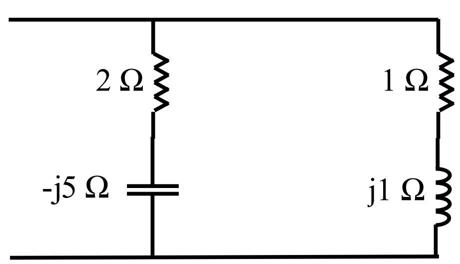
When θ is +ve, the impedance has inductive reactance, and the current through it lags the voltage across it. The power factor is, therefore, lagging. The following phasor diagram shows the current and the voltage across the impedance $|Z| \angle\theta$ in this case (i.e. when θ is positive).



When θ is -ve, the impedance has capacitive reactance, and the current through it leads the voltage across it. The power factor is, therefore, leading.

Problem.

Find the power factor of the following circuit.



Admittance (Y)

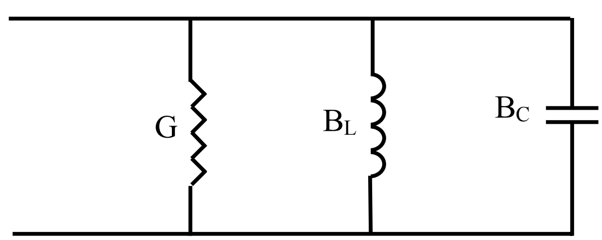
The reciprocal of impedance is called Admittance, and it is denoted by *Y*.

$Y = 1/Z$ $\qquad\qquad$ $Y = G + jB$

where, *G* is known as the conductance and *B* the susceptance.

$G = 1/R$ $\qquad\qquad$ $B = 1/X$

It is easier to analyze a parallel circuit when the circuit elements are expressed in admittances instead of impedances.



The total admittance of the above circuit is given by

$$Y = G + B_L + B_C$$
$$= G + 1/(j\omega L) + j\omega C$$

.

Real and Reactive Power

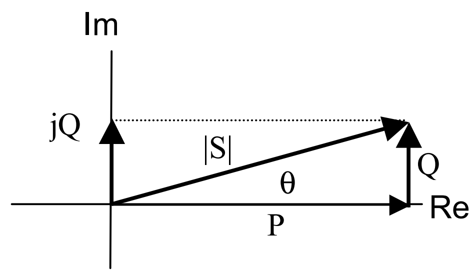
Complex Power, S is defined by the equation, $S = V\,I^*$

Since V and I are represented by complex numbers, the product V I* (i.e. complex power) is also a complex number. It can be expressed either in the polar form or the rectangular form.

$S = V\,I^*$

$= |S|\,\underline{/\theta^0}$ in the polar form.

$= P + jQ$ in the rectangular form.



Complex Plane

When expressed in the polar form, the magnitude |S| is known as the *Apparent Power* and the angle is the Power Factor angle. Apparent power is measured in Volt-amperes (VA). Power sources (generators) and transformers are generally rated in VA. The apparent power can be calculated using the equation,

$$|S| = |V| \,.\, |I|$$

When expressed in the rectangular form, the real part P is known as the *Real Power* or *Active Power*, and the imaginary part Q is known as the *Reactive Power*.

Real power is measured in Watts (W) and sometimes in horsepower (hp). It is the power that is capable of doing some useful work, such as, lighting, heating, rotating objects. The real or active power can be calculated using one of the following equations:

$$P = \mathrm{Re}\,(V.I^*)$$

$$P = |V| \,.\, |I| \,.\, \cos\theta$$

$$P = |I|^2 \,.\, R$$

where R is the resistance in which real power is consumed.

Real power is generated from power sources (generators). It is consumed in the resistances of a circuit in order to do some useful work, or is dissipated as power loss in a circuit.

Reactive power is measured in Volt-ampere-reactive (VAr). It is also known as hidden power. It is related to power quality. The reactive power can be calculated using one of the following equations:

$$Q = Re\ (V.I^*)$$

$$Q = |V|\ .\ |I|\ .\ \sin\theta$$

$$Q = |I|^2\ .\ X$$

where X is the reactance in which reactive power is generated or consumed.

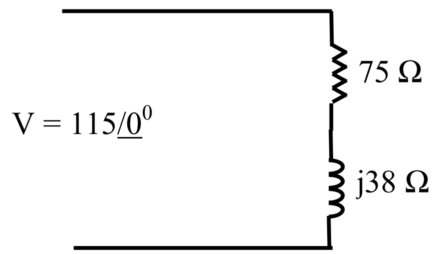
In a real sense, reactive power is neither generated nor consumed. It goes back and forth in every half cycle. It can be considered to be generated by a circuit reactance X in a positive half cycle and consumed by the same in the negative half cycle. It is generally assumed that reactive power is generated from power sources (generators) and capacitors/condensers (capacitive reactance X_C), and consumed in inductors (inductive reactance X_L) of a circuit. Power sources with inductive reactance (e.g. induction generator) consume reactive power.

Sign Convention for Power Analysis:

Power used or consumed:	+ ve
Power generated:	- ve

Problem.

Find the active power (P), reactive power (Q) and the power factor of the following circuit.



Power Source and Load

A power circuit can be considered to be composed of a ***source*** and ***load***.

Source – e.g. AC generator (synchronous, induction)

- generates active power, P is – ve
- may generate reactive power (Q is - ve) or use reactive power (Q is + ve)

Load – Circuit component that consumes real power, P is + ve

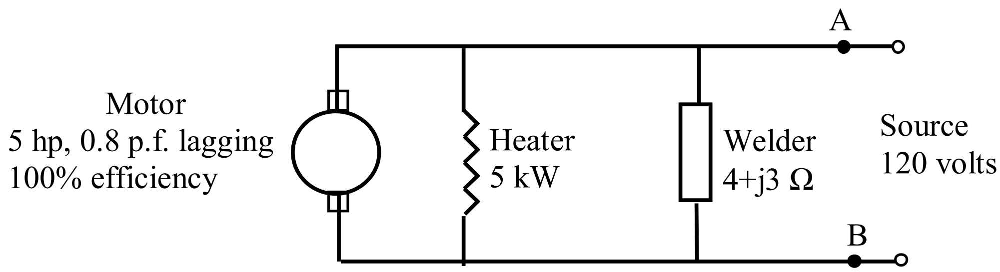
1. Resistive Load: e.g. heater, light bulbs
   power factor = 1, $\theta = 0$
   no reactive power, Q = 0

2. Inductive Load: e.g. motor, welder
   lagging power factor
   uses reactive power, Q = + ve

3. Capacitive Load: e.g. capacitor, synchronous motor (condenser)
leading power factor
generates reactive power, Q = - ve

Problem.

What is the power supplied to the combined load? What is the load power factor?
(Note: 1 hp = 746 W)



What value of capacitor must be connected to the terminals A-B in order to make the power factor equal to unity?

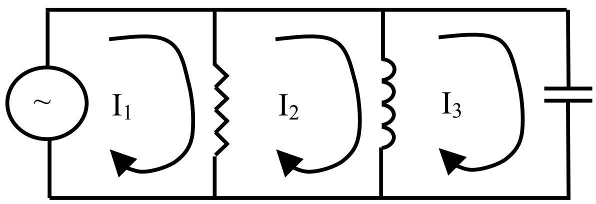
Power Factor Correction

Most industrial loads operate at a lagging power factor. Power consumption at a low power factor makes power utilization very inefficient. Power industries make it mandatory for industrial consumers to bring to their power factor close to unity, and this process is known as power factor correction. A capacitor (or condenser) of a proper value is connected in parallel to the loads for power factor correction. The following are the advantages of power factor correction.

- draws less current from the source
- less transmission losses (I^2R)
- less voltage drop in transmission, and therefore higher voltage at the load terminals
- less reactive power generated from source

Network Theorems

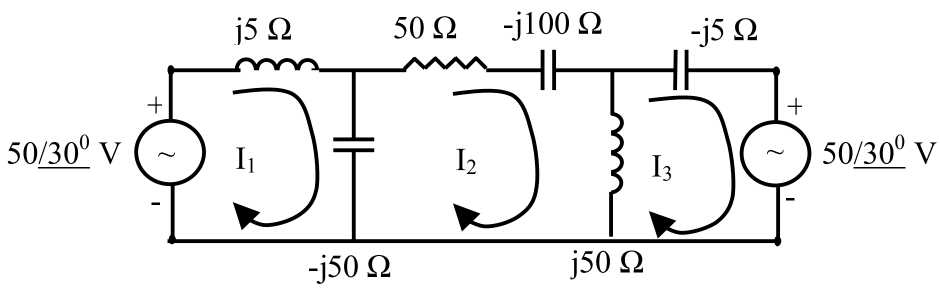
Mesh (Loop) Analysis



The Mesh Analysis can be applied for circuit analysis using the following procedure:

- assume loop current I_1 for Loop 1, I_2 for Loop 2, etc.
- apply KVL for each loop
- obtain 'n' no. of equations for 'n' no. of loops
- the loop currents are the unknown variables
- solve equations to determine loop currents
- obtain currents through each circuit elements
- apply Ohm's Law to calculate voltages
- use Power equations to calculate P, Q

The Loop Analysis method is illustrated by application to the following circuit.



Applying KVL at Loop 1:

$$-50\underline{/30^0} + I_1\, j5 + (I_1 - I_2).(-j50) = 0$$

or $$50\underline{/30^0} = -j45.I_1 + j50.I_2 \qquad \text{(Equation 1)}$$

Applying KVL at Loop 2:

$$(50 - j100)\, I_2 + (I_2 - I_3).(j50) + (I_2 - I_1).(-j50) = 0$$

or $$0 = j50.I_1 + (50 - j100)\, I_2 - j50.I_3 \qquad \text{(Equation 2)}$$

Applying KVL at Loop 3:

$$(-j5)\, I_3 + 50\underline{/30^0} + (I_3 - I_2).(j50) = 0$$

or $$-50\underline{/30^0} = -j50.I_2 + j45.I_3 \qquad \text{(Equation 3)}$$

There are three equations and three unknowns (i.e. I_1, I_2 and I_3). The unknowns can be calculated by solving the three equations. One method of solving linear equations is the matrix method. The matrix equation is shown below.

$$\begin{bmatrix} 50\underline{/30^0} \\ 0 \\ -50\underline{/30^0} \end{bmatrix} = \begin{bmatrix} -j45 & j50 & 0 \\ j50 & 50 - j100 & -j50 \\ 0 & -j50 & j45 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

The above matrix equation is in the form

$$[V] = [Z]\, [I]$$

where [Z] is the impedance matrix, [V] the voltage vector, and [I] is the current vector. [V] and [I] are column vectors, and [Z] is a square matrix.

The matrix equation can be obtained directly from the topology of the circuit if it has only voltage sources. The matrix equation for a circuit with ‘n’ loops, in which the current is considered to be in the clock-wise direction in each loop, is given by

$$\begin{bmatrix} V_1 \\ V_2 \\ \dots \\ \dots \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & -Z_{12} & \dots & \dots & -Z_{1n} \\ -Z_{21} & Z_{22} & \dots & \dots & -Z_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ -Z_{n1} & -Z_{n2} & \dots & \dots & Z_{1n} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \dots \\ \dots \\ I_n \end{bmatrix}$$

Where, Z_{ii} = self impedance (sum of all the impedances in Loop i)

Z_{ij} = impedance between Loop i and Loop j

I_i = current in Loop i (unknown variable)

V_i = voltage source in Loop I (-ve to +ve current flow is taken as +ve voltage)

The unknown currents in the matrix equation can be obtained from Cramer’s Rule.

$$I_k = \Delta_k / \Delta$$

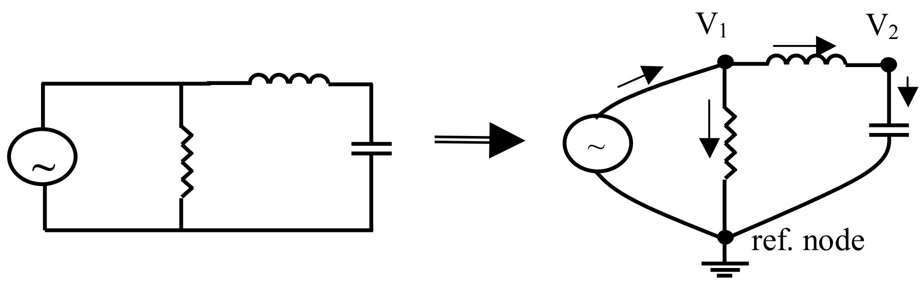
Where, Δ = determinant of the impedance matrix (i.e. |Z|)

and $\Delta_k = \Delta$ with the k^{th} column replaced by the voltage vector

From the matrix equation of the above circuit:

$$I_2 = \frac{\begin{vmatrix} -j45 & 50/\underline{30^0} & 0 \\ j50 & 0 & -j50 \\ 0 & -50/\underline{30^0} & j45 \end{vmatrix}}{\begin{vmatrix} -j45 & j50 & 0 \\ j50 & 50 - j100 & -j50 \\ 0 & -j50 & j45 \end{vmatrix}} = 0$$

Nodal Analysis

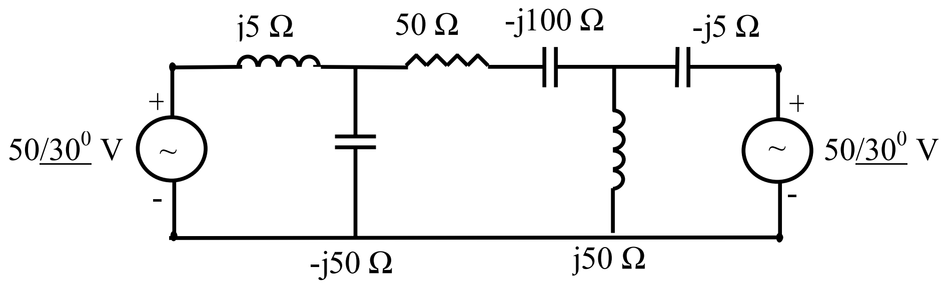


The Nodal Analysis can be applied for circuit analysis using the following procedure:

- identify all the nodes
- select a reference node (usually the node with the most no. of branches)
- assume voltage V_i (w.r.t . reference node) for Node i
- assume current direction in each branch
- apply KCL at each node
- obtain ‘n-1’ no. of equations for ‘n’ no. of nodes
- the node voltages are the unknown variables
- solve the equations to determine node voltages
- apply Ohm’s Law to calculate the currents
- use Power equations to calculate P, Q

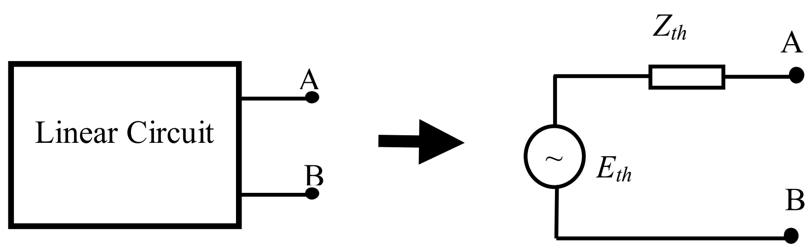
Problem.

Find the power dissipated in the 50 Ω resistor using both the Mesh (Loop) and Nodal analyses.



Thevenin's Theorem

Any linear two terminal network with sources can be replaced by an equivalent voltage source in series with an equivalent impedance.



Thevenin Voltage, E_{th} :

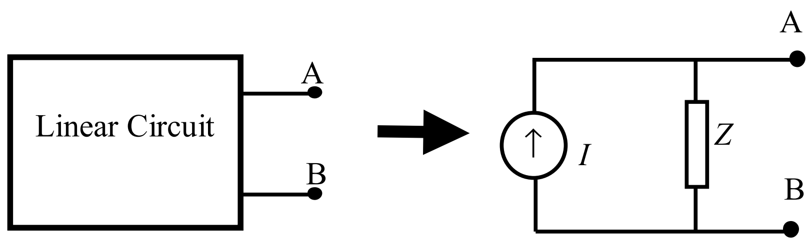
It is the voltage measured at the terminals A & B with nothing connected to the external circuit.

Thevenin Impedance, Z_{th} :

It is the impedance at the terminals A & B with all the sources reduced to zero.
i.e. voltage sources short circuited (0 volts),
and current sources open circuited (0 amps).

Norton's Theorem

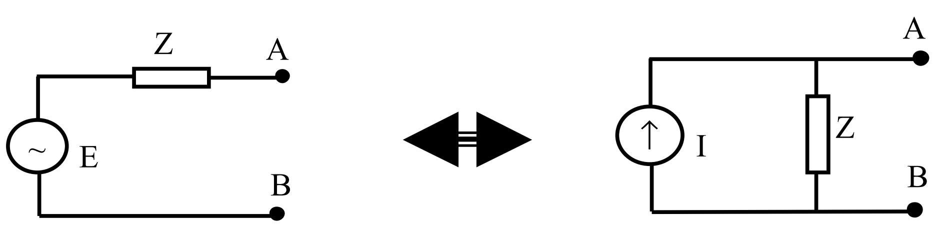
Any linear two terminal network with sources can be replaced by an equivalent current source in parallel with an equivalent impedance.



Current source *I* is the current which would flow between the terminals if they were short circuited.

Equivalent impedance *Z* is the impedance at the terminals (looking into the circuit) with all the sources reduced to zero.

Thevenin Equivalent **Norton Equivalent**



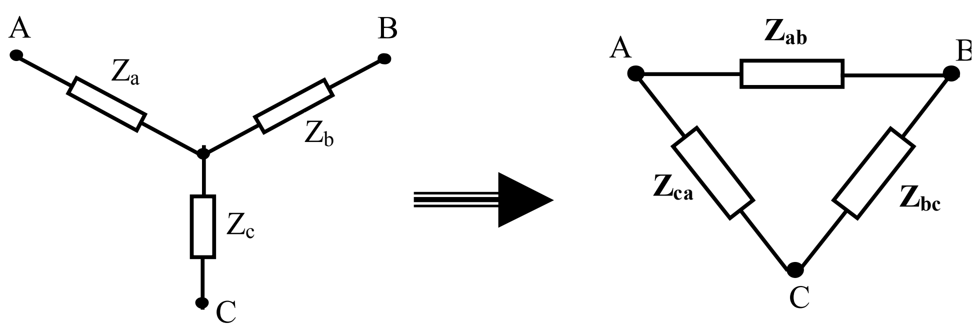
E = IZ **I = E / Z**

Note: equivalence is at the terminals with respect to the external circuit.

Wye-Delta (Y-Δ) Transformation

A Y-connected circuit can be represented by an equivalent delta-connected circuit as shown in the figure below. This transformation becomes very useful when dealing with three phase systems which are connected either in wye or delta. Some parts of a three phase systems may be delta connected, and other may be wye connected. Wye-delta (or Delta-wye) transformations can be used to convert the entire circuit in one configuration (either wye or delta).

A wye-connected circuit can be transformed to an equivalent delta circuit using the following equations:



$$Z_{ab} = (Z_a.Z_b + Z_b.Z_c + Z_c.\ Z_a) / Z_c$$

$$Z_{bc} = (Z_a.Z_b + Z_b.Z_c + Z_c.\ Z_a) / Z_a$$

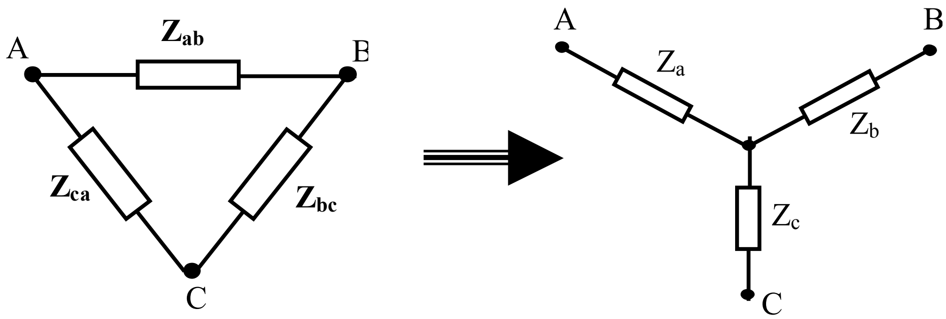
$$Z_{ca} = (Z_a.Z_b + Z_b.Z_c + Z_c.\ Z_a) / Z_b$$

$$Z_{\Delta} = \frac{\sum(\text{product of } Z_Y \text{ taken in pairs})}{\text{the opposite } Z_Y}$$

The equivalence is at the three terminals A, B and C with respect to the external circuit. This means that any parameters calculated external to the points A-B-C in the equivalent circuit is also true for the original circuit. Any parameters calculated internal to A-B-C in the equivalent circuit, however, have no meaning in the original circuit.

Delta-Wye (Δ-Y) Transformation

A delta-connected circuit can be transformed to an equivalent wye circuit using the following equations:



$$Z_a = (Z_{ab} . Z_{ca}) / (Z_{ab} + Z_{bc} + Z_{ca})$$

$$Z_b = (Z_{ab} . Z_{bc}) / (Z_{ab} + Z_{bc} + Z_{ca})$$

$$Z_c = (Z_{ca} . Z_{bc}) / (Z_{ab} + Z_{bc} + Z_{ca})$$

$$Z_Y = \frac{\text{(product of adjacent } Z_\Delta)}{\text{sum of all 3 } Z_\Delta}$$

The equivalence is at the three terminals A, B and C with respect to the external circuit.

Superposition Theorem

If a linear circuit has 2 or more sources acting jointly, we can consider each source acting separately (independently) and then superimpose the 2 or more resulting effects.

Procedure:

- Analyze the circuit considering each source separately
- To remove sources, short circuit V sources and open circuit I sources
- For each source, calculate the voltages and currents in the circuit
- Sum the voltages and currents

Superposition Theorem is very useful when analyzing a circuit that has 2 or more sources with different frequencies.

Power circuits are often excited by periodic waveforms that are non-sinusoidal. Application of power electronic devices in power systems often causes a distortion of sinusoidal voltage supply, resulting in non-sinusoidal periodic voltage and current waveforms in a power circuit. Superposition Theorem becomes important when analyzing these types of circuits excited by non-sinusoidal periodic waveforms. This is because non-sinusoidal periodic waveforms can be decomposed into a number of sinusoidal waveforms of different frequencies, known as harmonics.

Non-sinusoidal Periodic Waveforms

A non-sinusoidal periodic waveform, f(t) can be expressed as a sum of sinusoidal waveforms. This is known as Fourier Analysis.

Fourier Series is expressed as:

$$f(t) = a_0 + \sum (a_n \text{ Cos } n\omega t) + \sum (b_n \text{ Sin } n\omega t)$$

where,

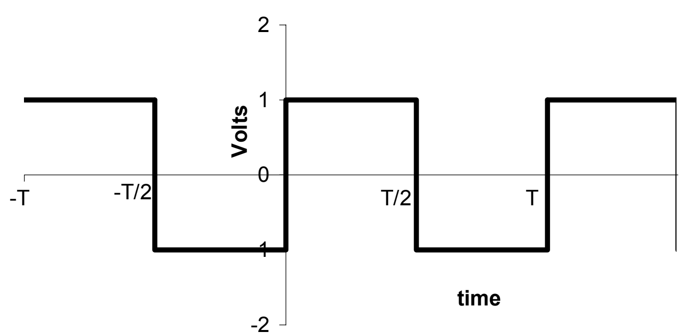
a_0 = average over one period (dc component) $= \frac{1}{T}\int_0^T f(t).dt$

$a_n = \frac{2}{T}\int_0^T f(t).\text{Cos}(n\omega t).dt$ for $n > 0$

$b_n = \frac{2}{T}\int_0^T f(t).\text{Sin}(n\omega t).dt$ for $n > 0$

A square waveform is an example of a non-sinusoidal periodic waveform. It can be expressed mathematically as,

$$f(t) = 1 \quad \text{for} \quad 0 \le t \le T/2$$
$$= -1 \quad \text{for} \quad T/2 \le t \le T$$



The above square waveform can be expressed as a sum of sinusoidal waveforms using the Fourier analysis.

First, the Fourier Series constants are evaluated for the square waveform.

a_0 = average over one period = 0

$$a_n = \frac{2}{T}\int_0^T f(t).Cos(n\omega t).dt = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t).Sin(n\omega t).dt = \frac{4}{n\pi}(1-\cos\frac{n\pi}{2})$$

A square waveform can, therefore, be expressed as:

$$f(t) = \frac{4}{\pi}\ (Sin\ \omega t + \frac{1}{3}\ Sin\ 3\omega t + \frac{1}{5}\ Sin\ 5\omega t + \ldots..)$$

A linear circuit with non-sinusoidal periodic sources can be analyzed using the Superposition Theorem. The following procedure can be used to carry out the analysis:

1. Express the non-sinusoidal function by its Fourier series
2. Use Superposition Theorem and calculate voltages and currents for each harmonic
3. Calculate the final voltages and currents by summing up all the harmonics
4. Equations for RMS values and Power:

$$V_{rms} = \sqrt{(V_0^2 + V_{1rms}^2 + V_{2rms}^2 + V_{3rms}^2 + \ldots)}$$

$$= \sqrt{(V_0{}^2 + \frac{1}{2}v_1{}^2 + \frac{1}{2}\ v_2{}^2 + \frac{1}{2}v_3{}^2 + \ldots)} \quad \text{in term of peak values}$$

$$I_{rms} = \sqrt{(I_0{}^2 + \frac{1}{2}i_1{}^2 + \frac{1}{2}\ i_2{}^2 + \frac{1}{2}i_3{}^2 + \ldots)} \quad \text{in term of peak values}$$

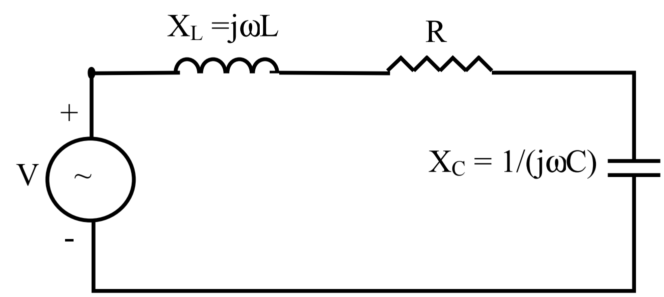
$$P = V_0 I_0 + \frac{1}{2} v_1 i_1 \cos\theta_1 + \frac{1}{2} v_2 i_2 \cos\theta_2 + ..$$

$$= |I_{rms}|^2 R$$

$$Q = \frac{1}{2} v_1 i_1 \sin\theta_1 + \frac{1}{2} v_2 i_2 \sin\theta_2 + ..$$

Series and Parallel Resonance

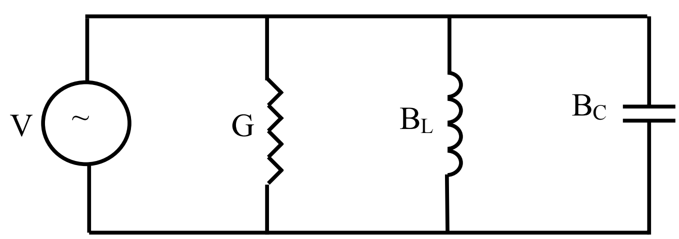
Consider a series circuit,



$$Z = R + j\omega L - j/\omega C$$

$$= R + j(\omega L - 1/\omega C)$$

Consider a parallel circuit,



$$Y = G + B_L + B_C$$

$$= G + 1/(j\omega L) + j\omega C$$

$$= G + j(\omega C - 1/\omega L)$$

The imaginary parts of both equations are zero when

$$\omega L = 1/(\omega C)$$

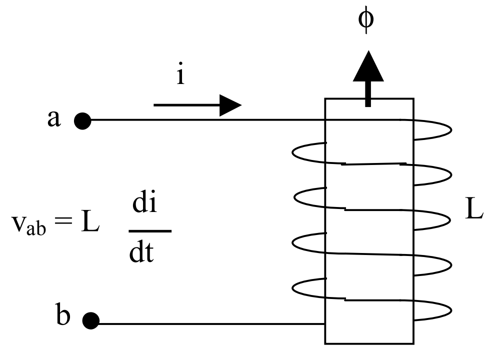
or $$\omega_0 = \frac{1}{\sqrt{LC}}$$

ω_0 is known as the resonant angular frequency.

Resonant frequency, $$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The frequency at which the impedance Z of a series circuit or the admittance Y of a parallel circuit is purely real, is called the resonant frequency.

Coupled Circuits



Consider a coil around a magnetic core. If a current i flows through the coil, a flux ϕ is generated in the core. Magnetic flux is measured in weber (wb). The magnitude of the flux ϕ is given by the equation,

$$\phi = \frac{1}{R_m} N\,i$$

where:

N = number of turns in the coil

R_m = constant known as reluctance

The reluctance depends on the magnetic path of the flux, and is measured in Ampere-turns per weber (AT/wb). It is mathematically expressed as,

$$R_m = \frac{l}{\mu . A} \quad \text{AT/wb}$$

where μ, A and l are the permeability, cross-section area, and length of the magnetic path respectively.

The direction of the flux ϕ can be determined by the Right-Hand Rule.

Fingers curled around coil – direction of current

Thumb – direction of flux

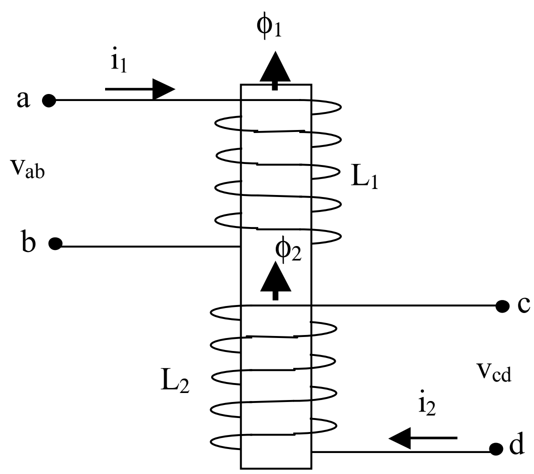
Other important terminology and equations useful in magnetic circuits are:

Flux Density, $B = \dfrac{\phi}{A}$ wb/m^2

Magnetizing Force, $H = \dfrac{N.I}{l}$ AT/m

$B = \mu H$

Coupled Circuits are circuits that affect each other by mutual magnetic fields. The following figure shows two circuits magnetically coupled with each other.



The flux ϕ_2 generated by current i_2 in Coil 2 induces a voltage in Coil 1. The voltage across Coil 1 is given by the following equation (using Kirchoff's Voltage Law), in which the second term is the voltage induced from the other circuit, i.e. Coil 2.

$$v_{ab} = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

Similarly, The flux ϕ_1 generated by current i_1 in Coil 1 induces a voltage in Coil 2. The voltage across Coil 2 is given by the following equation:

$$v_{cd} = L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}$$

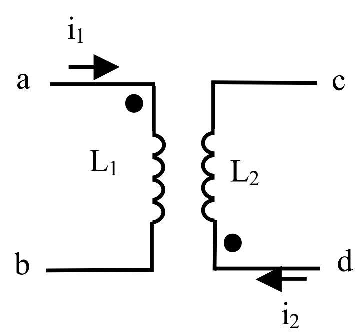
where:

L_1 and L_2 are self inductances of Coil 1 and Coil 2 respectively, and

M is the mutual inductance between Coil 1 and Coil 2.

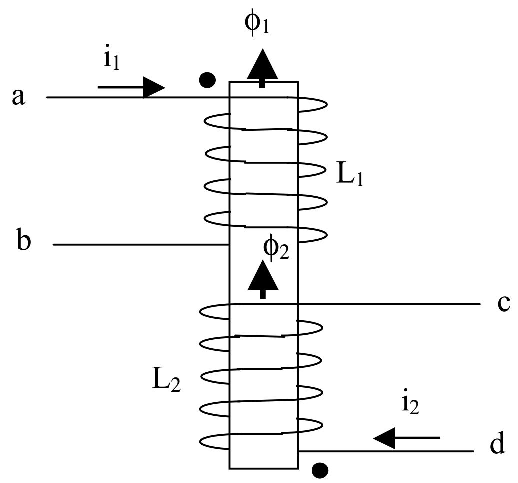
The sign between the two terms in the above equations is + or - depending on whether the fluxes add or oppose each other. If both the fluxes ϕ_1 and ϕ_2 are in the same direction, the fluxes add each other, and therefore, a +ve sign is used in the above equation. If the two fluxes are in opposite directions, a –ve sign is used. It is, therefore, important to determine the direction of the induced fluxes in the analysis of coupled circuits.

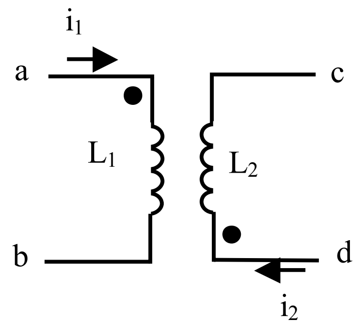
The above figure of coupled circuits shows the direction of current through the coil around the core. Coupled circuits are, however, represented by the following circuit diagram.



It can be seen from the above figure that the orientation of the coil winding is not shown in the circuit diagram. A *Dot Convention* is used instead, and a dot is shown at an end of each coil to provide the information on how the coils are wound around the core with respect to each other. The above coupled-circuits can be analyzed by obtaining an equation for each circuit, and determining the sign + or – for the equations using the dot convention.

The Dot Convention





Since a circuit diagram (on the right) does not show the direction of the coil winding, this information must be provided by placing dots at the proper ends of each coil. This is done in two steps.

Step 1: Place a dot at one end of a coil.

Step 2: Place the other dot at the proper end of the other coil, so that currents entering the dots produce fluxes that add each other.
In the above example, the currents entering the dots produce upward fluxes.

The + or – sign for the coupled circuit equations are determined by the following dot convention:

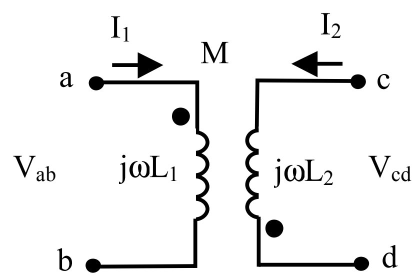
- Use (+) sign in the equations if both currents enter the dotted terminals (or the undotted terminals).

- Use (-) sign in the equations if one current enters a dotted terminal and the other current enters an undotted terminal.

A current *i* entering a dotted terminal in one coil induces a voltage $M\frac{di}{dt}$ with a positive polarity at the dotted terminal of the other coil. *M* is the mutual inductance between the two coils.

Phasor Method for Coupled Circuit Analysis

If input signals are sinusoidal waveforms, coupled circuits can be easily analyzed using the Phasor Method. The phasor representation of the circuit diagram is shown below.



The following equations are obtained for each circuit using Kirchoff's Voltage Law:

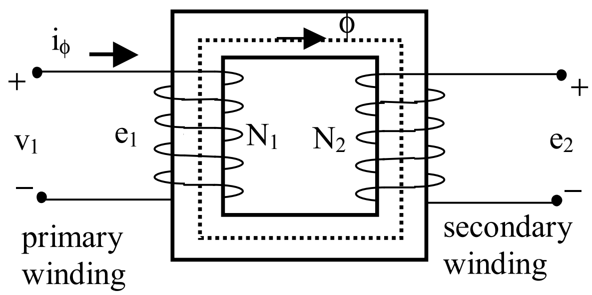
$V_{ab} = (j\omega L_1)\, I_1 \pm (j\omega M)\, I_2$

$V_{cd} = (j\omega L_2)\, I_2 \pm (j\omega M)\, I_1$

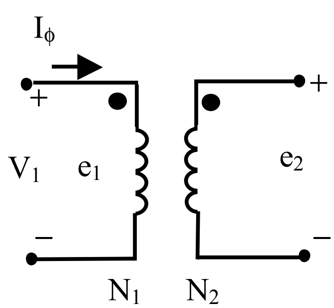
The polarity + or – is determined from the Dot Convention. (In this example, the sign is – ve.)

Transformer

A transformer is an energy transfer device that can basically be viewed as coupled circuits. The three main parts of a transformer are the primary coil (or winding), secondary winding, and the magnetic core around which the coils are wound. The following diagram shows the main parts of a transformer and its principle of operation. When an AC voltage v_1 is applied to the primary terminals, a current i_ϕ flows through the primary winding, which generates a flux ϕ that flows through the magnetic core and induces an electromotive force (emf) e_2 across the secondary winding.



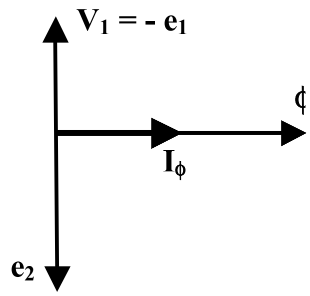
A transformer can be represented by the following circuit diagram (which is a circuit representation for coupled circuits).



The emf e_2 induced in the secondary winding can be determined by Faraday's Law of electromagnetic induction (i.e. *the induced emf is directly proportional to the rate of change of flux linkage*).

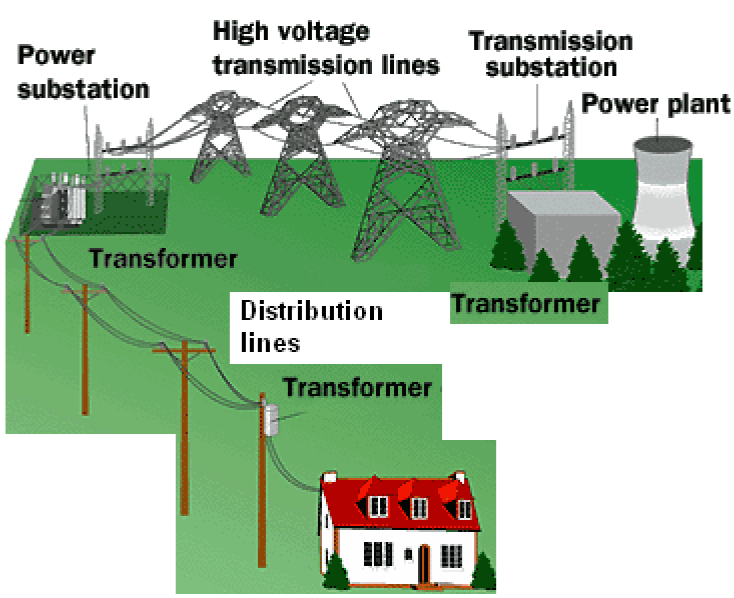
$$e_2 = N_2 \frac{d\varphi}{dt} \quad \rightarrow \quad \text{Faraday's Law}$$

The induced emf lags the inducing flux by 90^0. N_1 and N_2 are the number of turns in the primary and secondary windings respectively. The phasor diagram is shown below.

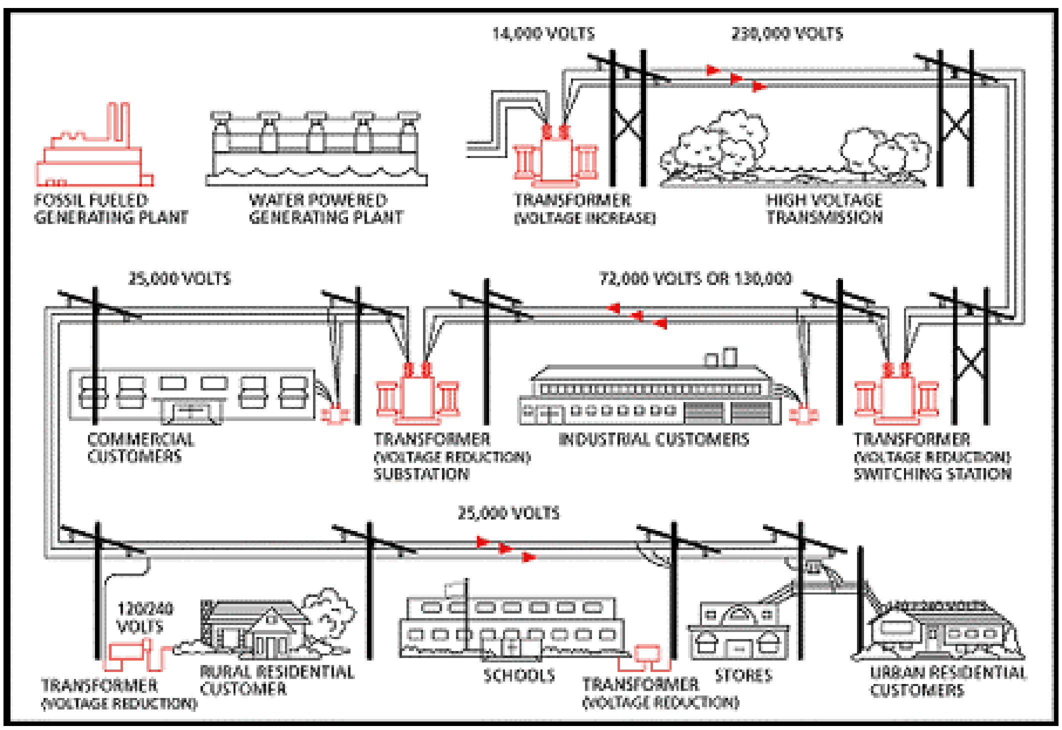


There are two main purposes for using transformers. The first is to convert the energy on the primary side to a different voltage level on the secondary side. This is accomplished by using differing number of turns on primary and secondary windings. The second purpose is to isolate the energy source from the destination.

Power/distribution transformers are used to convert voltage levels in electrical power transmission and distribution. A typical power system consists of four parts: generation (or source), transmission, distribution, and consumption (or load). The following figure shows a basic power system.



Power generated by the source (generator) is generally transmitted through long distances before it is consumed by the load. Power is transmitted at very high voltages to minimize power loss in transmission. Power loss is given by the square of the current multiplied by the resistance of the wire. A power transformer steps the voltage up after power is generated from the source. Increasing the voltage will decrease the current by the same ratio, and therefore, lower the power loss in transmission. As the transmitted power approaches close to community areas, the voltage is stepped down in several stages using transformers. This is done for safety reasons. Power is then distributed to consumers (load points) through the distribution line at a relatively lower voltage. Power is finally stepped down to operating voltages by distribution transformers before reaching each customer (load). The following figure shows a simplified diagram of a power grid similar to that of Sask Power.

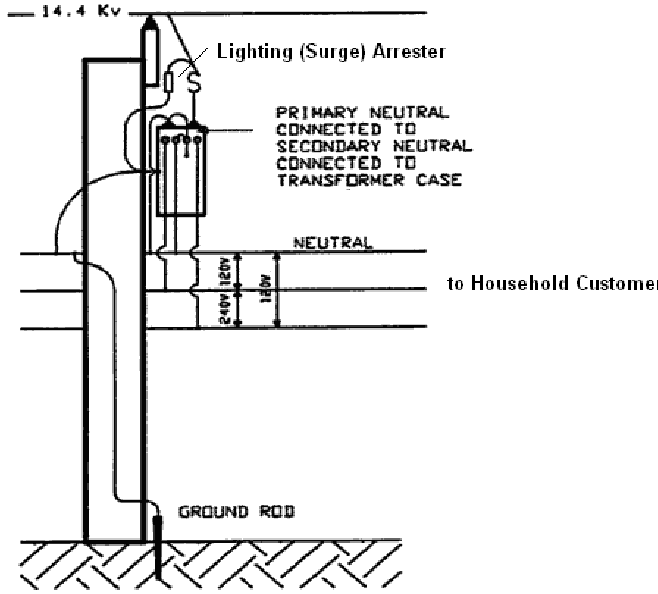


The distribution transformers which are located at the load point are either single-phase or three-phase. Almost all of the other transformers used in a power system are three-phase. Three single-phase transformers are sometimes used instead of one single-phase transformer. The following figures show single-phase and three-phase transformers.

Pole-mounted Single-phase Transformer

Three-phase Transformer

The power supply to a household usually comes through a pole mounted drum type transformer in this part of the world. The following figure shows how the transformer is connected.

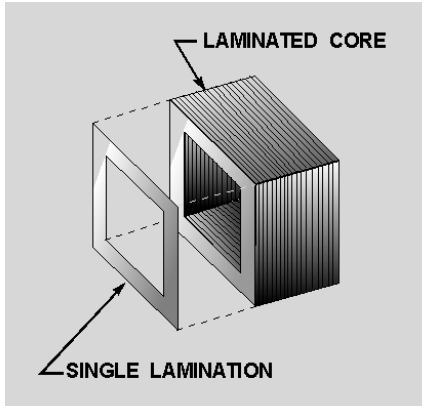


Transformer Construction

The manufacturing process of a transformer includes the following basic steps:

- Coil Winding
- Core Assembly
- Core-Coil Assembly
- Tank-up
- Accessories Mounting and Finishing

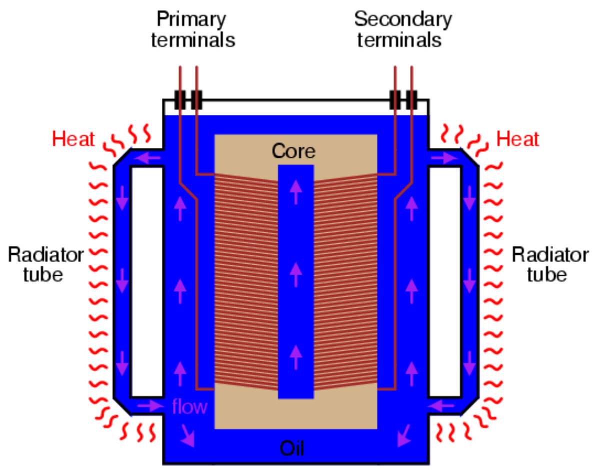
High voltage and low voltage coils are wound on formers of proper diameter and length. Thin sheets of steel are stacked together to form a laminated core as shown below:



The vertical sides of the core are known as the limbs. The top horizontal side, known as the yoke, is removed to insert the coils into the limbs. The low voltage (LV) coil is first placed on the insulated core limbs. Insulating block of specified thickness and number are placed both at the top and bottom of the LV coil. Cylinder made out of corrugated paper is provided over the LV Coil. The high voltage (HV) coil is placed over the cylinder. The top yoke is fixed in position. Primary and secondary windings are connected as per the requirements. The core-coil assembly is then placed in a tank as shown below:

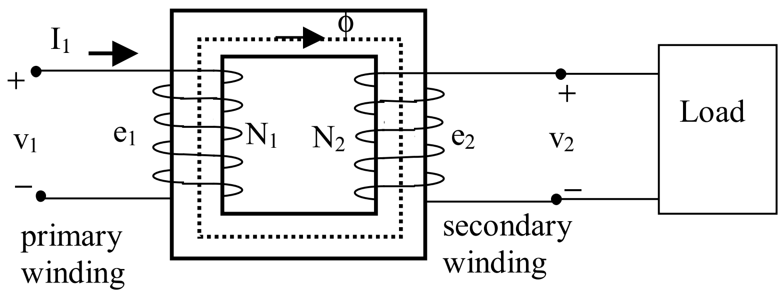
Pure filtered transformer oil is filled in the tank to immerse the assembly. Connections of primary and secondary coil ends to the terminal bushings are made. Transformer tap changer and protection accessories (e.g. Buchholz relay, Conservator, Breather, temperature indicator, etc.) are installed and the tank is closed.

Most power transformers have their core and windings submerged in an oil bath to transfer heat, muffle noise, and also to displace moisture which would otherwise compromise the integrity of the winding insulation. Heat-dissipating "radiator" tubes on the outside of the transformer case provide a convective oil flow path to transfer heat from the transformer's core to ambient air.



Ideal Transformer

A good way of understanding the analysis of a transformer is to start with an ideal transformer. The analysis then becomes very simple. Gradually, more complexities are added to the initial analysis to represent an actual transformer operation. An ideal transformer has no leakage flux and no losses.



Ideal Transformer has No leakage flux:

When an AC voltage v_1 is applied to the primary terminals, the primary current i_1 generates a flux ϕ. The flux ϕ induces an emf e_1 in the primary winding which is given by:

$$e_1 = N_1 \frac{d\varphi}{dt}$$

Since an ideal transformer has no leakage flux, the same flux ϕ passes through the core and links the secondary winding inducing an emf e_2 given by

$$e_2 = N_2 \frac{d\varphi}{dt}$$

The ratio of the two emf's, $\frac{e_1}{e_2} = \frac{N_1}{N_2} = a$ (turns ratio)

Ideal Transformer has No Losses:

The instantaneous power in the primary of the transformer = $v_1 \cdot i_1$

and the instantaneous power in the secondary of the transformer = $v_2 \cdot i_2$

Since an ideal transformer has no losses, all the energy from the primary is transferred to the secondary winding.

$$v_1 i_1 = v_2 i_2$$

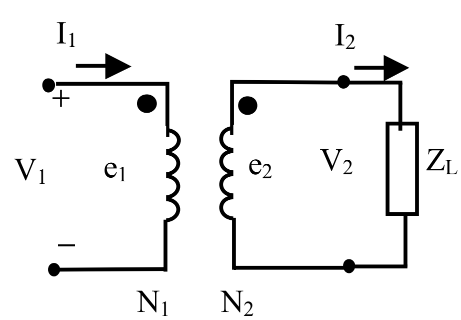
Since an ideal transformer has no voltage drops in the windings,

$$v_1 = -e_1 \text{ and } v_2 = -e_2$$

Therefore, $$e_1 i_1 = e_2 i_2$$

Or, $$\frac{i_2}{i_1} = \frac{e_1}{e_2} = \frac{N_1}{N_2} = a$$

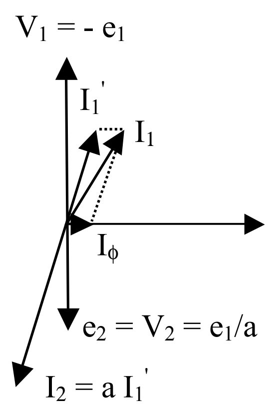
When a load is connected to the secondary winding, the following process occurs:



- a secondary current I_2 is drawn by the load
- I_2 generates a flux that opposes the mutual flux ϕ (Lenz's Law: effect opposes the cause)
- Reduction in mutual flux ϕ would reduce e_1
- but since source voltage V_1 is constant, and $V_1 = - e_1$, the mutual flux ϕ must remain constant
- i.e. the primary winding must draw an additional current I'_1 from the source to neutralize the demagnetizing effect from the secondary.

  Primary current $I_1 = I'_1 + I_\phi$

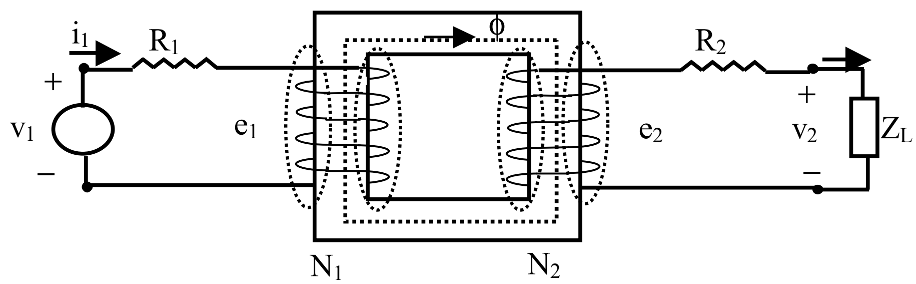
The phasor diagram is shown below.



Actual Transformer and Losses

The following additional parameters must be considered in an actual transformer.

- resistances in primary (R_1) and secondary (R_2) windings
- leakage reactances in primary (X_1) and secondary (X_2) windings
- voltage drops in both windings due to leakage impedances Z_1 and Z_2
- losses



The following types are power losses occur in an actual transformer:

- **Copper Loss**

Copper loss occurs in both the primary and secondary windings. This is the power dissipated as heat when the current flows through the resistances of the primary and secondary windings.

$$\text{Cu loss in primary} = I_1^2.R_1$$
$$\text{Cu loss in secondary} = I_2^2.R_2$$

The resistances are generally assumed to be constant in transformer analysis. As the load connected to the secondary of the transformer varies, the current also varies, and therefore, the copper loss varies as well. The copper loss is, therefore, a function of the loading condition of a transformer.

- **Iron Loss or Core Loss**

The iron loss is the power dissipated as heat due to the movement of magnetic flux in the iron core of the transformer. It is represented by the following equation in an equivalent circuit of a transformer:

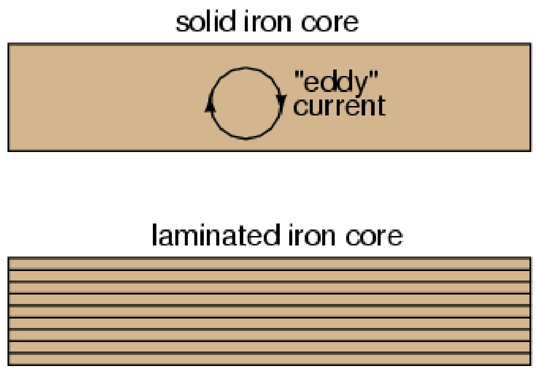
$$\text{Iron or Core Loss} = I_c^2.R_c$$

where R_c is the equivalent core loss resistance, and I_c the current component through R_c.

The above equation is not generally used to determine the core loss. It is determined from the *Open Circuit Test* of a transformer. Core loss depends on the voltage and frequency. In practice, transformers are designed to work at a particular frequency and voltage. The core loss is therefore assumed to be constant in transformer analysis. Iron or core loss mainly constitutes of *Eddy Current Loss* and *Hystereis Loss*.

Eddy Current Loss:

The alternating flux in the iron core induces emf in the core, due to which, currents circulate in closed loops within the core material. These circulating currents are known as eddy currents which generate heat in the iron core as they flow against the resistance in the core.

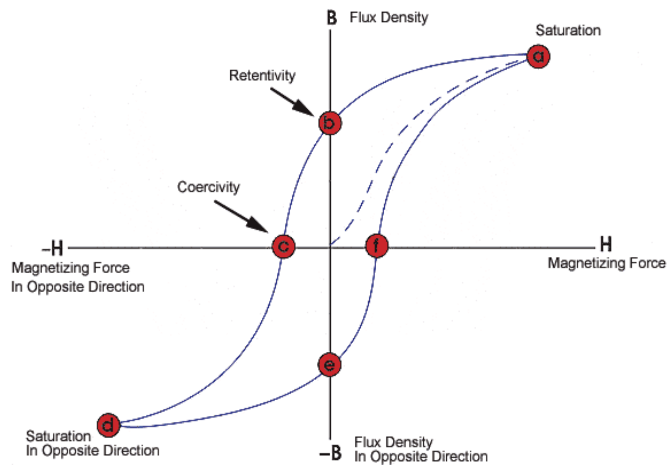


In order to minimize eddy current losses, laminated iron core is used in a transformer instead of a solid iron core as shown in the figures above. The core is made up of many layers of thin iron sheets with insulating material (silica sheets or varnish) in between. This results in significant increase in resistance to the current path, and consequently significant reduction in eddy currents.

Hysteresis Loss:

All ferromagnetic materials tend to retain some degree of magnetization after exposure to an external magnetic field. This tendency to stay magnetized is called "hysteresis," and it takes a certain investment in energy to overcome this opposition to change every time the magnetic field produced by the primary winding changes polarity (twice per AC cycle). In other words, power is consumed to move around the magnetic dipoles in the core material as the direction of the magnetic flux in the core changes every cycle. The energy consumed in doing so is dissipated as heat.

Hysteresis loss is proportional to the volume of the core and the area of the hysterisis loop. The following figure shows a typical hysteresis loop for a magnetic material. This type of loss can be reduced through good core material selection (choosing a core alloy with low hysteresis, as evidenced by a "thin" B/H hysteresis curve), and designing the core for minimum flux density (large cross-sectional area).



A hysteresis loop shows the relationship between the induced magnetic flux density B and the magnetizing force H. It is often referred to as the B-H loop. The loop is generated by measuring the magnetic flux B of a ferromagnetic material while the magnetizing force H is changed.

A ferromagnetic material that has never been previously magnetized or has been thoroughly demagnetized will follow the dashed line as H is increased. As the line demonstrates, the greater the amount of current applied (H+), the stronger the magnetic field in the component (B+). At point "a" almost all of the magnetic domains are aligned and an additional increase in the magnetizing force will produce very little increase in magnetic flux. The material has reached the point of magnetic saturation.

When H is reduced back down to zero, the curve will move from point "a" to point "b." At this point, it can be seen that some magnetic flux remains in the material even though the magnetizing force is zero. This is referred to as the point of retentivity on the graph and indicates the remanence or level of residual magnetism in the material. As the magnetizing force is reversed, the curve moves to point "c", where the flux has been reduced to zero. This is called the point of coercivity on the curve. The force required to remove the residual magnetism from the material, is called the coercive force or coercivity of the material.

As the magnetizing force is increased in the negative direction, the material will again become magnetically saturated but in the opposite direction (point "d"). Reducing H to zero brings the curve to point "e." It will have a level of residual magnetism equal to that achieved in the other direction. Increasing H back in the positive direction will return B to zero. Notice that the curve did not return to the origin of the graph because some force is required to remove the residual magnetism. The curve will take a different path from point "f" back the saturation point where it with complete the loop.

Transformer Rating

A transformer's power transfer capacity is rated in Volt-amperes (VA). The rated operating voltages for the primary and secondary windings are also provided by the manufacturer. The winding with the higher voltage rating is known as the HV winding and the other winding as the LV winding. The ratio of the two rated voltages is the turns ratio of the transformer. If the LV winding is used as the primary winding, the transformer is used as a step-up transformer. On the other hand, the transformer is used in the step-down mode if the HV winding is used as the primary winding.

The rated current is the maximum current the transformer can withstand continuously, and is also known as the full load current. When rated currents flow in a transformer, it is said to be fully loaded. The rated current in a single phase transformer can be determined by dividing the rated VA by the rated voltage.

Rated Primary Current, $I_{1(rated)} = VA_{(rated)} / V_{1(rated)}$

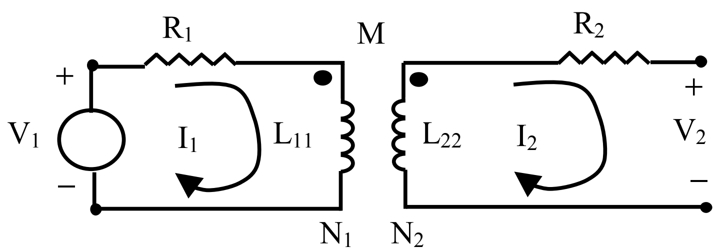
Rated Secondary Current, $I_{2(rated)} = VA_{(rated)} / V_{2(rated)}$

The rated frequency for power and distribution transformers are wither 50 Hz or 60 Hz. The manufacturer usually provides the values of other design parameters of a transformer.

It should be noted that the operating voltages and currents in a transformer can be quite different from the rated values. The operating values, however, should not exceed the rated values. Transformers are selected by engineers for particular applications, and the ratings are chosen in such a way that they are usually close to the operating values. The operating current can be quite different from the rated current in an application where there is a variable load connected to the transformer.

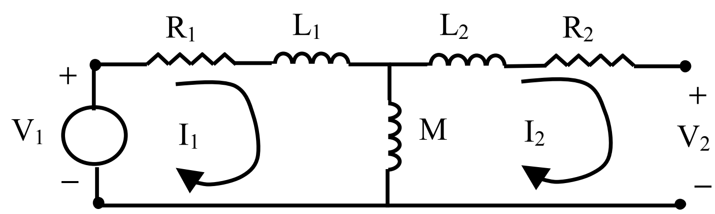
Transformer Equivalent Circuit

A transformer is an important component of a power circuit. A transformer must be represented by its equivalent circuit in order to carry out the analysis of a power circuit. The equivalent circuit represents the inductively coupled circuits by a conductively connected circuit. The equivalence is in terms of loop equations.



inductively coupled circuits

equivalent to



conductively connected circuit

Loop Equations for the inductively coupled circuits:

KVL at Loop 1: $-V_1 + R_1I_1 + j\omega L_{11}I_1 - j\omega M\, I_2 = 0$

KVL at Loop 2: $R_2I_2 + V_2 + j\omega L_{22}I_2 - j\omega M\, I_1 = 0$

The loop equations are:

$$\begin{bmatrix} V_1 \\ -V_2 \end{bmatrix} = \begin{bmatrix} R_1 + j\omega L_{11} & -j\omega M \\ -j\omega M & R_2 + j\omega L_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Loop Equations for the conductively connected circuit:

Let $L_1 = L_{11} - M$ and $L_2 = L_{22} - M$

The loop equations are:

$$\begin{bmatrix} V_1 \\ -V_2 \end{bmatrix} = \begin{bmatrix} R_1 + j\omega(L_1 + M) & -j\omega M \\ -j\omega M & R_2 + j\omega(L_2 + M) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Substituting the values of L_1 and L_2

$$\begin{bmatrix} V_1 \\ -V_2 \end{bmatrix} = \begin{bmatrix} R_1 + j\omega L_{11} & -j\omega M \\ -j\omega M & R_2 + j\omega L_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Since the loop equations for the inductively coupled circuits and the conductively connected circuit shown above are the same, the two circuit diagrams are equivalent in terms of loop equations.

The primary and secondary windings of a transformer have different number of turns. The turns ratio *a* of a transformer must be considered when deriving its equivalent circuit.

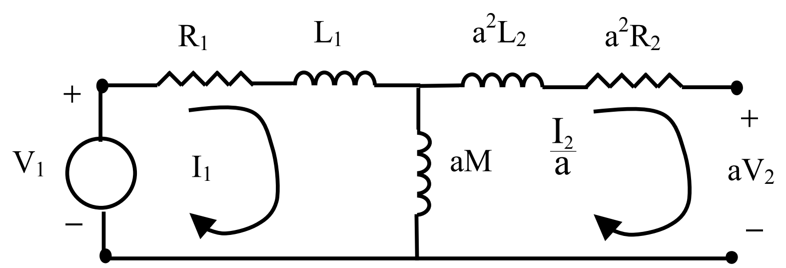
$$\text{Turns ratio, } \frac{N_1}{N_2} = a$$

Consider the conductively connected circuit with the following substitutions:

$M \rightarrow a.M$ $\quad$ $L_2 \rightarrow a^2.L_2$ $\quad$ $R_2 \rightarrow a^2.R_2$

$V_2 \rightarrow a.V_2$ $\quad$ $I_2 \rightarrow \dfrac{I_2}{a}$

Then $L_1 = L_{11} - aM$ and $L_2 = L_{22} - M/a$



KVL at Loop 1: $-V_1 + R_1I_1 + j\omega L_1I_1 + j\omega aM\,(I_1\text{-}I_2/a) = 0$

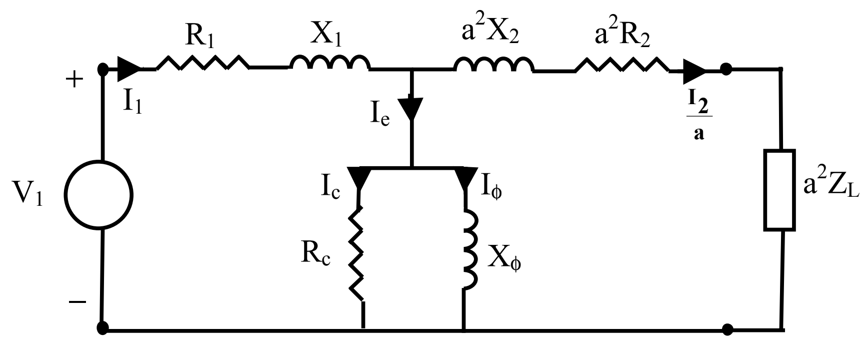
KVL at Loop 2: $j\omega aM\,(I_2/a - I_1) + j\omega a^2L_2.I_2/a + j\omega a^2R_2.I_2/a + aV_1 = 0$

The loop equations are:

$$\begin{bmatrix} V_1 \\ -V_2 \end{bmatrix} = \begin{bmatrix} R_1 + j\omega L_{11} & -j\omega M \\ -j\omega M & R_2 + j\omega L_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Since the loop equations are the same, the above conductively connected circuit is also equivalent to the inductively coupled circuit which has a turns ratio *a*.

A transformer can therefore be represented by the equivalent circuit shown below.



Where,

R_1, R_2= primary, secondary winding resistance

X_1, X_2= primary, secondary leakage reactance

I_1, I_2= primary, secondary current

Z = load impedance

I_e = excitation current

I_ϕ = magnetizing current

X_ϕ = magnetizing reactance

R_c = core loss resistance (equivalent resistance contributing to core loss)

I_c = core loss equivalent current

The above equivalent circuit is said to be referred to the primary side. In this case, the primary parameters (R, X, V, I) remain the same as the actual transformer parameters, whereas the secondary parameters are changed due to the turns ratio.

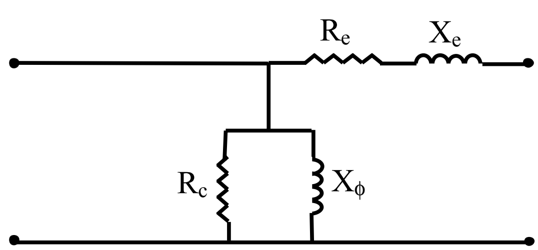
Problem.

A 50 kVA, 2400/240 V, 60 Hz distribution transformer has a leakage impedance of (0.72 + j0.92) Ω in the high voltage (HV) side and (0.0070 + j0.0090) Ω in the low voltage (LV) winding. At rated voltage and frequency, the admittance of the shunt branch of the equivalent circuit is (0.324 – j2.24)x10^{-2} mho when viewed from the LV side. Draw the circuit:

a) viewed from the LV side
b) viewed from the HV side

Approximate Equivalent Circuits

A transformer is often represented by an approximate equivalent circuit instead of an actual equivalent circuit. This is done to simplify the analysis without causing significant error in the results. The following equivalent circuit, in which both the primary and secondary impedances are shown on the same side, is often used in transformer circuit analysis.



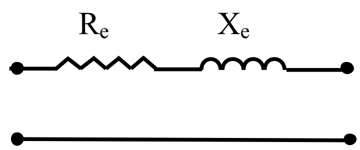
Where,

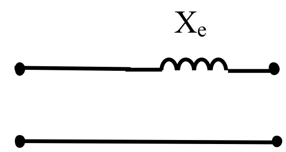
$R_e = R_1 + a^2R_2$

$X_e = X_1 + a^2X_2$

and the circuit parameters are referred to the primary side.

Other equivalent circuits used in power system studies are shown below.





The equivalent circuit parameters are determined from the *Short Circuit Test* and *Open Circuit Test* of the transformer.

R_e and X_e $\rightarrow$ obtained from Short Circuit Test

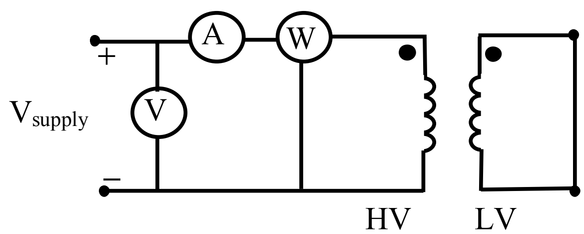
R_c and X_ϕ $\rightarrow$ obtained from Open Circuit Test

Short circuit and open circuit tests are carried out by energizing one winding, while short-circuiting and open-circuiting the other winding respectively. A wattmeter, a voltmeter and an ammeter are connected on the supply side to monitor the real power, voltage and current.

Short Circuit Test

In a short circuit test, one winding is short circuited, and the other winding is energized to allow rated current to flow in the transformer.

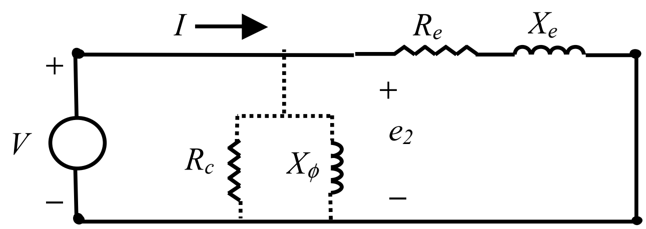
In a short circuit condition, only 2 – 12 % of the rated voltage is required at the source to obtain rated current in the transformer. It is advantageous to connect the metering instruments (wattmeter, voltmeter and ammeter) and energize the HV side, and short the LV side of the transformer. By using this configuration, less expensive and lower current rated instruments can be used since the rated current is lower on the HV side. A better resolution and control of the supply voltage is also obtained by energizing the HV side.



The following procedure is used in a short-circuit test:

- Connect the meters on the HV side as shown
- Short circuit the LV side
- Energize the HV side with a variable voltage source and increase voltage gradually to get rated current reading on the Ammeter
- Take the voltage V, current I (rated value) and power P readings from meters

Since a relatively small voltage is applied at the primary, the emf e_2 induced in the secondary is also very small. The core flux that induces e_2 must therefore be very small. The core loss is negligible compared to the overall power loss of the transformer, and the excitation current is negligible compared to the load current at such a small value of the core flux. The shunt branch (consisting of the R_c and X_ϕ parameters) of the transformer equivalent circuit can therefore be neglected. The equivalent circuit is shown below. Since the meters are placed on the HV side, the readings indicate the values of the HV side parameters. The equivalent circuit is, therefore, referred to the HV side.



Equivalent circuit referred to HV side

All the power fed into the transformer is dissipated as heat in the transformer windings. The wattmeter reading P, therefore, gives the copper loss of the transformer.

$$P \;=\; \text{copper loss} \;=\; I^2 R_e \quad \rightarrow \quad R_e = \frac{P}{I^2}$$

The ratio of the voltage and the current gives the total leakage impedance Z_e of the transformer.

$$Z_e = \frac{V}{I} \quad \rightarrow \quad X_e = \sqrt{Z_e^2 - R_e^2}$$

The equivalent circuit parameters R_e and X_e are thus obtained from the short circuit test. It should be noted that the values of these parameters are referred to the HV side.

The secondary winding resistance referred to the primary is approximately equal to the primary winding resistance in power and distribution transformers. The individual primary and secondary winding resistance can therefore be calculated from the following equation:

$$R_1 \approx a^2 R_2 \approx \frac{R_e}{2}$$

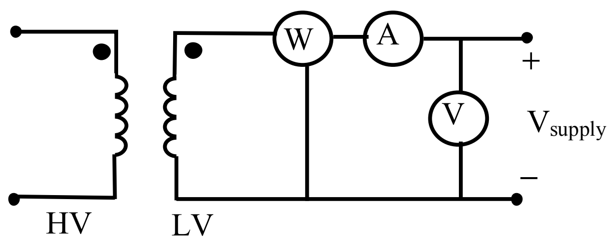
Similarly the individual primary and secondary winding leakage reactance can be obtained by the following relation.

$$X_1 \approx a^2 X_2 \approx \frac{X_e}{2}$$

Open Circuit Test

In an open circuit test, one winding of the transformer is open circuited, and the other winding is energized with the rated voltage.

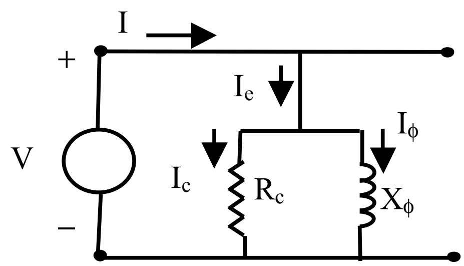
In an open circuit condition, there is no current in the secondary winding, and the only current in the circuit is the excitation current which flows through the primary winding. The excitation current is only 2 – 6 % of the rated current. Since rated voltage must be applied to the transformer in this test, it is advantageous to energize the LV side and connect the metering instruments (wattmeter, voltmeter and ammeter) while leaving the HV terminals open. By using this configuration, less expensive and lower voltage rated instruments can be used since the rated voltage is lower on the LV side.



The following procedure is used in a short-circuit test:

- Connect the meters on the LV side as shown
- Open circuit the HV side
- Energize the LV side with the rated voltage
- Take the voltage *V* (rated value), current *I* and power *P* readings from meters

There is no copper loss in the secondary winding as there is no current flowing in an open circuit condition. Since the excitation current flowing through the primary winding is very small, the voltage drop and the copper loss in the primary can be neglected. The series branch (consisting of the R_e and X_e parameters) of the transformer equivalent circuit can therefore be neglected. The equivalent circuit is shown below. Since the meters are placed on the LV side, the readings are values of the LV side parameters. The equivalent circuit is, therefore, referred to the LV side.



Equivalent circuit referred to LV side

Almost all of the power fed into the transformer is dissipated as heat in the transformer iron core. The wattmeter reading *P*, therefore, gives the core loss of the transformer.

$$P \quad = \quad \text{core or iron loss} \; = \; \frac{V^2}{R_c} \quad \rightarrow \; R_c = \frac{V^2}{P}$$

The voltage across R_c and X_ϕ can be assumed to be equal to the source voltage V. The current I_c through the resistor is in phase with the voltage *V*, and the current I_ϕ through the inductor lags the voltage *V* by 90^0. The current I_ϕ therefore lags the current I_ϕ by 90^0. The phasor sum of I_c and I_ϕ is equal to the excitation current I_e (which is also the current *I* measured by the ammeter).

$$I_e = I \, , \qquad\qquad I_c = \frac{V}{R_c}$$

$$I_\phi = \sqrt{I_e^{\,2} - I_c^{\,2}} \qquad \rightarrow \qquad X_\phi = \frac{V}{I_\varphi}$$

The equivalent circuit parameters R_c and X_ϕ are thus obtained from the open circuit test. It should be noted that the values of these parameters are referred to the LV side.

Problem.

Find the equivalent circuit of a 50 kVA, 2400/240 V transformer. The following readings were obtained from short circuit and open circuit tests:

Test	V-meter (V)	A-meter (A)	W-meter (W)
Short Circuit	48	20.8	617
Open Circuit	240	5.41	186

If rated voltage is available at the load terminals, calculate the transformer efficiency at

a) full load at 0.8 p.f. lagging

b) 60% load at 0.8 p.f. lagging

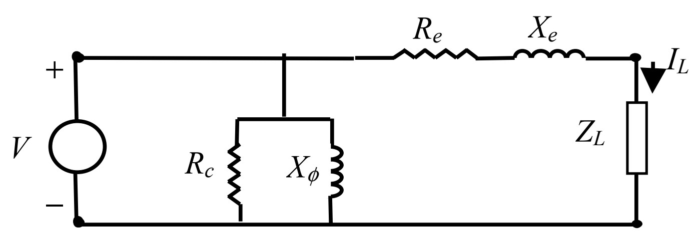
Transformer Efficiency

A transformer is an energy transfer device. Due to the inherent losses (copper and core losses) in a transformer, the power output is always less than the power input in the transformer. The transformer efficiency is defined as,

Efficiency, η = Power Output / Power Input

Power Input = Power Output + Copper losses + Core losses

The following figure shows a transformer connected to an AC source of voltage V and load Z_L. The load draws a current I_L.



The copper loss varies with load current and is obtained by the equation,

Copper loss $= I_L^2 R_e$

The core (iron) loss depends on the voltage and frequency. It is determined from the Open Circuit test and is usually assumed to be a constant for practical purposes.

Transformer efficiency is maximum when the copper loss is equal to the core loss.

The output power is the real power consumed by the load connected to the secondary terminals. It can be determined from the power rating of the load, or by using an appropriate power equation. If R_L is the load resistance in the above circuit, the power output can be calculated by the equation,

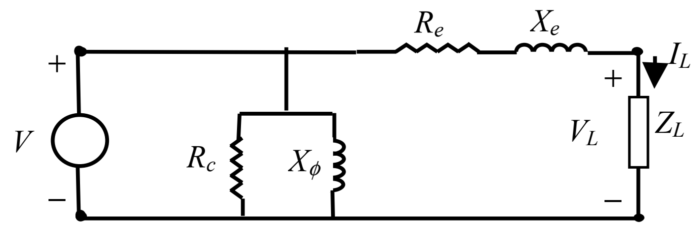
$$\text{Power output} = I_L^{2} R_L$$

The efficiency of a transformer depends on its design. Power transformers are designed to have maximum efficiency at full load. This is because these transformers are usually operated at rated capacity.

Distribution transformers, on the other hand carry a widely varying load. A distribution transformer is located close to the power consumer (e.g. a household load), and the level of power consumption can vary widely throughout the days and at different seasons. These transformers are designed to have maximum efficiency at about 60% of full load. Since these transformers are always energized despite load levels, they are designed to have low core losses.

Transformer Voltage Regulation

Electrical loads, such as lighting loads, appliances and different types of electrical gadgets, require a specified constant voltage for normal operation. It is therefore important to have a constant voltage at the load terminals of a transformer.



The voltage at the load terminals is equal to the source voltage minus the voltage drop in the transformer windings.

$$V_L = V - I_L\,(R_e + jX_e)$$

The load voltage changes with the load current even though a constant voltage is supplied by the source in the primary terminals. As the load current varies throughout the day, the available voltage at the load terminals varies as well. The voltage regulation of a transformer specifies the amount of maximum variation in voltage due to changes in current (from zero to full load).

Rated voltage is normally applied to the primary of a transformer. Assume a constant rated voltage is applied to the primary at no load condition.

The voltage drop in the windings in zero since there is no load current. The load voltage (or the secondary voltage, V_{sec}) is therefore equal to the source voltage. Since rated voltage is applied to the primary,

$$V_{sec} = \text{rated voltage}$$

If a load is connected at the load terminals, the load (or secondary) voltage will vary as the load current varies. An electrical load is usually resistive or inductive in nature. As the load current increases, the voltage drop in transformer windings will increase, and therefore, the secondary voltage will decrease. However, if the load is capacitive in nature, the secondary voltage will increase as the load current increases.

The source (primary) voltage can be varied with the load current throughout the day in order to maintain a constant rated voltage at the load (secondary) terminals. The voltage required at the primary to obtain the rated voltage at the secondary can be calculated at different loading conditions as follows:

At no load,

primary voltage required, V_{pr} = rated voltage

At a certain load,

primary voltage required V_{pr} = rated voltage + V_{drop} in transformer

Voltage regulation is the change in primary voltage required to keep the secondary voltage constant from no load to full load, expressed as a percentage of rated primary voltage. The power factor of the load has a big effect on voltage regulation. Voltage regulation (*VR*) of a transformer can be calculated using the following equation:

$$VR = \frac{|V_{pr-FL}| - |V_{rated}|}{|V_{rated}|}$$

where $|V_{pr\text{-}FL}|$ is the primary voltage required to maintain rated voltage at the secondary during full load.

Problem.

A 50 kVA, 2400/240 V, 60 Hz transformer has a leakage impedance of (1.42+j1.82) ohms on the HV side.

1. Calculate the voltage regulation at
   a) 0.8 p.f. lagging
   b) unity p.f.
   c) 0.5 p.f. leading

2. The transformer is used to step down the voltage at the load end of a feeder whose impedance is 0.3 + j1.6 ohm. The voltage at the sending end of the feeder is 2400 V. Find the voltage at the secondary terminals of the transformer when the connected load draws rated current at a p.f. of 0.8 lagging.

3. What will be the current on the low voltage side if a short circuit occurs at the load point?

Three Phase Systems

Bulk power generation and transmission systems are three-phase (3-ϕ) systems. Generation and transmission of electrical power are more efficient in 3-ϕ systems than in 1-ϕ systems.

Three phase power generation has the following advantages over single phase:

- steady power (1-ϕ power is fluctuating)
- more efficient conversion of mechanical power to electrical power (3 times power with additional armature windings and slightly more torque)

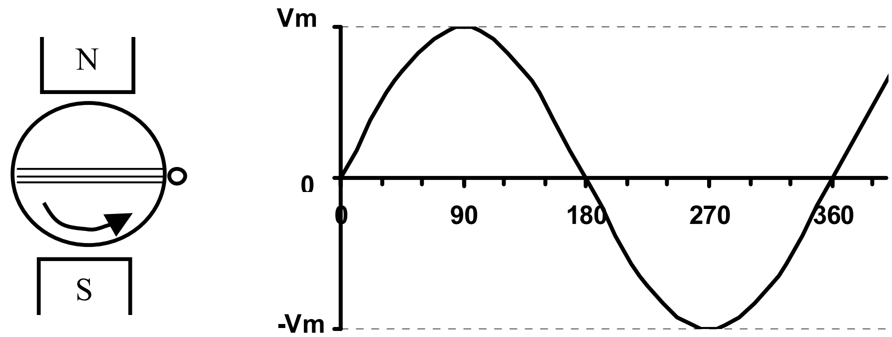
Three phase power transmission has the following advantages over single phase transmission:

- less conducting material required to transmit power (delta transmission – no return conductor)
- 3-ϕ transformers are more efficient

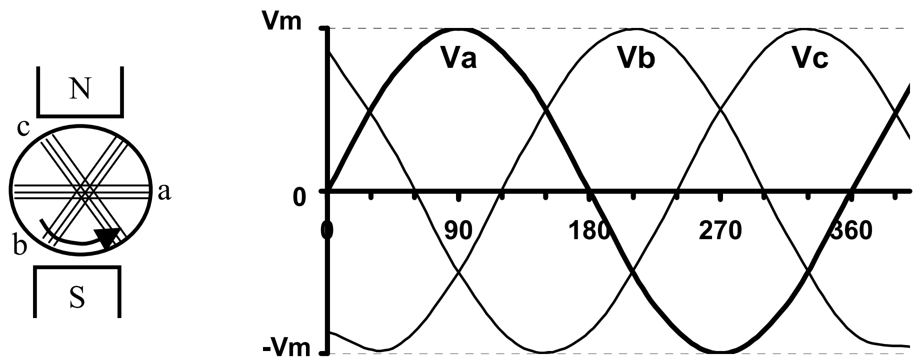
In terms of power consumption, 3-ϕ machines start and run more efficiently that single phase.

Most lighting loads, heating loads and small motors require 1-ϕ supply, whereas, industrial loads, and larger motors require 3- ϕ supply.

1-ϕ Power Generation:



3-ϕ Power Generation:



Phase Sequence is a-b-c

Phase Sequence: the order in which the voltages of the individual phases reach their maximum values

Voltages in the three phases (120^0 out of phase):

Phase a $\rightarrow$ $v_a = V_m \sin \omega t$

Phase b $\rightarrow$ $v_b = V_m \sin (\omega t - 120^0)$

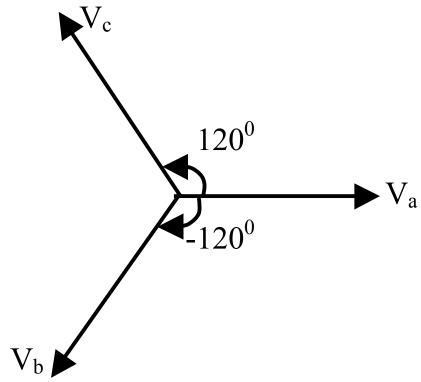
Phase c $\rightarrow$ $v_c = V_m \sin (\omega t - 240^0) = V_m \sin (\omega t + 120^0)$

In the phasor form,

$V_a = \frac{V_m}{\sqrt{2}} \underline{/0^0}$

$V_b = \frac{V_m}{\sqrt{2}} \underline{/-120^0}$

$V_c = \frac{V_m}{\sqrt{2}} \underline{/120^0}$



3 Phases: a-b-c, R-Y-B, L1-L2-L3

Types of 3-ϕ Systems:

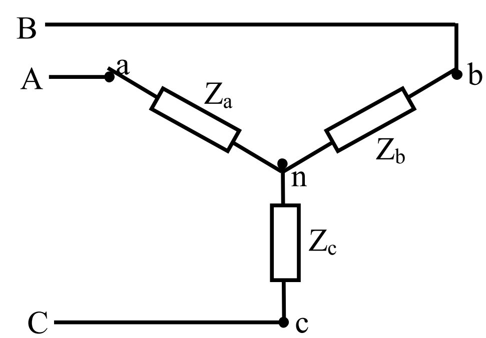
- Balanced Systems:
  - 3 phases (V & I) are equal in magnitude and 120^0 out of phase
  - can be analyzed considering only one phase
- Unbalanced Systems

Types of 3-ϕ Connections:

- Y (wye or star) connection
- Δ (delta) connection

Balanced Wye (Y) System

The following circuit shows a balanced three phase wye system. The following equations show the relation between the phase and line voltages and currents in the balanced Y system.



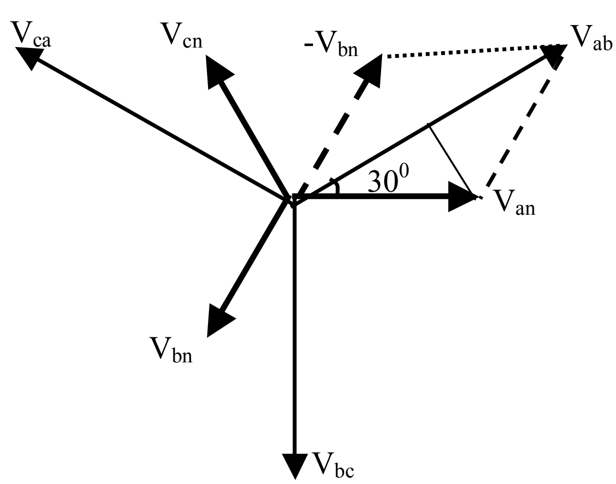
Phase voltages, V_p: $|V_{an}| = |V_{bn}| = |V_{cn}|$

Phase currents, I_p: $|I_{an}| = |I_{bn}| = |I_{cn}|$

Line voltages, V_L: $|V_{ab}| = |V_{bc}| = |V_{ca}|$

Line currents, I_L: $|I_a| = |I_b| = |I_c|$

The order in which each phase voltage or current reaches the peak value is known as the phase sequence. The normal phase sequence is a-b-c. The phasor diagram is shown below.



From phasor diagram:

$$V_{ab} = V_{an} + V_{nb}$$
$$= V_{an} - V_{bn}$$

$$\text{Cos } 30^0 = (½|V_{ab}|) / |V_{an}| = \sqrt{3}/2$$

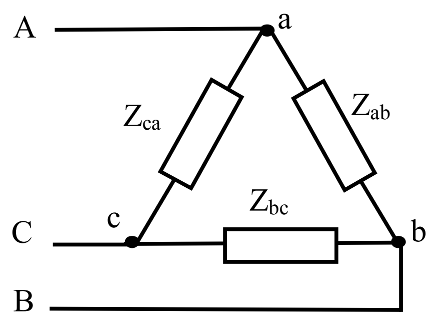
i.e. $|V_{ab}| = \sqrt{3}\,|V_{an}|$ and V_{ab} leads V_{an} by 30^0

$$|V_{line}| = \sqrt{3}\,|V_{phase}|$$

$$I_{line} = I_{phase}$$

Balanced Delta (Δ) System

The following circuit shows a balanced three phase wye system.



The following equations show the relation between the phase and line voltages and currents in the balanced Y system.

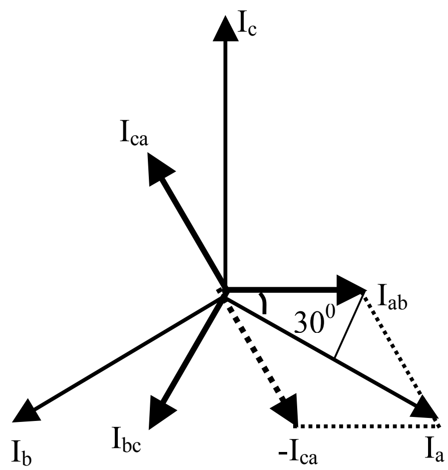
Phase voltages, V_p: $|V_{ab}| = |V_{bc}| = |V_{ca}|$

Phase currents, I_p: $|I_{ab}| = |I_{bc}| = |I_{ca}|$

Line voltages, V_L: $|V_{ab}| = |V_{bc}| = |V_{ca}|$

Line currents, I_L: $|I_a| = |I_b| = |I_c|$

The normal phase sequence is a-b-c. The phasor diagram is shown below.



From phasor diagram:

$I_a = I_{ab} + I_{ac} = I_{ab} - I_{ca}$

$\text{Cos } 30^0 = (\frac{1}{2}|I_a|) / |I_{ab}| = \sqrt{3}/2$

i.e. $|I_a| = \sqrt{3}\ |I_{ab}|$ and I_a lags I_{ab} by 30^0

$V_{line} = V_{phase}$

$|I_{line}| = \sqrt{3}\ |I_{phase}|$

In balanced 3-ϕ systems:

Y system $\rightarrow$ $|V_L| = \sqrt{3}\,|V_p|$, $I_L = I_p$, $V_{ab(line)}$ leads $V_{an(ph)}$ by 30^0

Δ system $\rightarrow$ $V_L = V_p$, $|I_L| = \sqrt{3}\,|I_p|$, $I_{a(line)}$ lags $I_{ab(ph)}$ by 30^0

Power Factor of a 3-ϕ load $\rightarrow$ $\cos\theta$

where, θ is the angle between the phase current and the phase voltage.

3-ϕ power = 3 x Per phase power

$$P\ (3\text{-}\phi) = 3(|V_p|.|I_p|.\cos\theta) = \sqrt{3}.|V_L|.|I_L|.\cos\theta$$

$$Q\ (3\text{-}\phi) = 3(|V_p|.|I_p|.\sin\theta) = \sqrt{3}.|V_L|.\ |I_L|.\ \sin\theta)$$

$$S\ (3\text{-}\phi) = 3(|V_p|.|I_p|) = \sqrt{3}.|V_L|.\ |I_L|)$$

3 identical loads connected in delta consume 3 times more power than the case when they are connected in wye.

Balanced three phase systems can be analyzed in terms of single phase systems. A single phase equivalent circuit can be drawn to represent the three phase system. The per phase quantities are determined and all calculations are done in per phase. The final result can be converted back to three phase circuit if required.

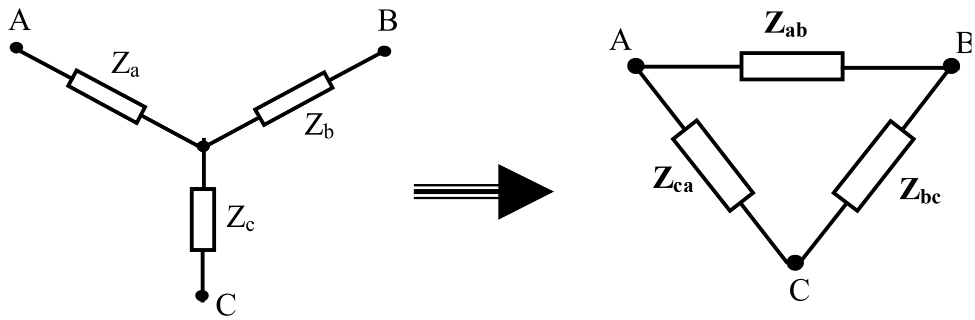
Problem.

A balanced 3-ϕ, line-to-line voltage of 220 V is applied to a balanced 3-ϕ load. The 3-ϕ load consists of 3 identical loads (each with an impedance of $6 + j8\ \Omega$)

(a) connected in Y
(b) connected in Δ

Calculate the line current, power consumed per phase and the total power consumed for both cases. Draw complete phasor diagrams for each case.

Wye-Delta (Y-Δ) Transformation



$Z_{ab} = (Z_a.Z_b + Z_b.Z_c + Z_c.\ Z_a) / Z_c$

$Z_{bc} = (Z_a.Z_b + Z_b.Z_c + Z_c.\ Z_a) / Z_a$

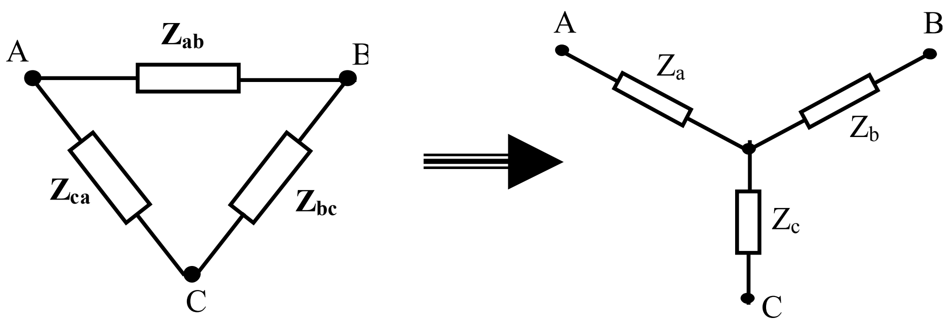
$Z_{ca} = (Z_a.Z_b + Z_b.Z_c + Z_c.\ Z_a) / Z_b$

$$Z_{\Delta} = \frac{\sum(\text{product of } Z_Y \text{ taken in pairs})}{\text{the opposite } Z_Y}$$

For balanced load, $Z_a = Z_b = Z_c = Z_Y$

Therefore, $Z_{\Delta} = 3.Z_Y$

Delta-Wye (Δ-Y) Transformation



$$Z_a = (Z_{ab}.\ Z_{ca}) / (Z_a + Z_b + Z_c)$$

$$Z_b = (Z_{ab}.\ Z_{bc}) / (Z_a + Z_b + Z_c)$$

$$Z_c = (Z_{ca}.\ Z_{bc}) / (Z_a + Z_b + Z_c)$$

$$Z_Y = \frac{\text{(product of adjacent } Z_\Delta)}{\text{sum of all 3 } Z_\Delta}$$

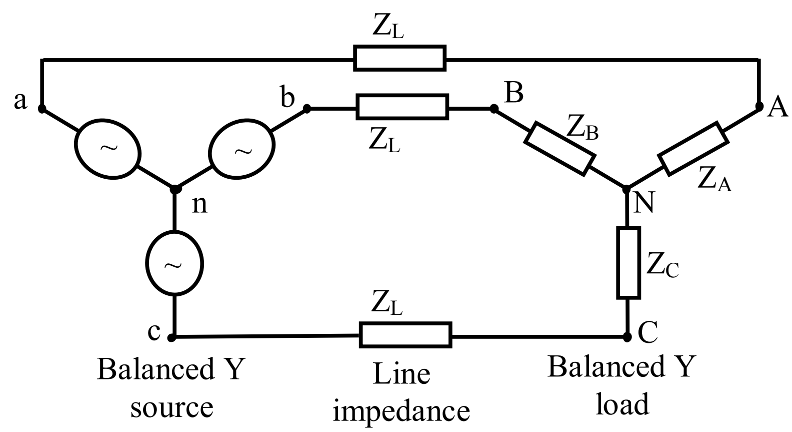
For balanced load, $Z_{ab} = Z_{bc} = Z_{ca} = Z_\Delta$

Therefore, $Z_Y = Z_\Delta / 3$

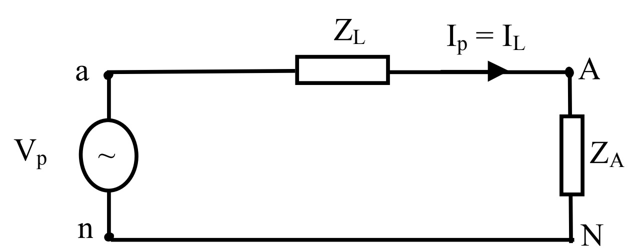
Balanced 3-ϕ Source and Load

In these types of three phase systems, both the source and the load are balanced systems. If both the systems are connected in Y, the system is known a Y-Y system. If the source is connected in Y and the load is connected in Δ, we get a Y-Δ system. The other two types of balanced three phase source and load systems are Δ - Δ and Δ -Y systems.

Y-Y System

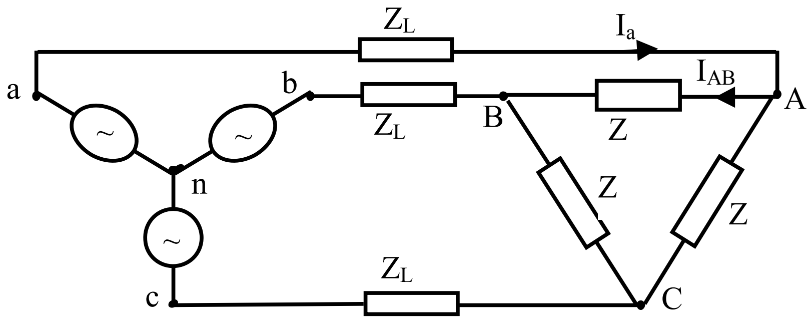


A per phase equivalent circuit for the above three phase circuit is shown below.

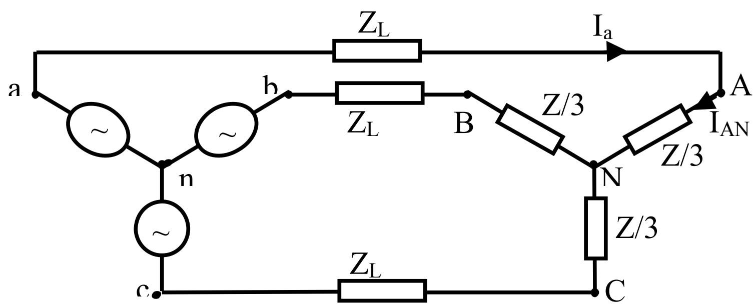


The above circuit can easily be analyzed as a single phase circuit, and the necessary parameters can be calculated.

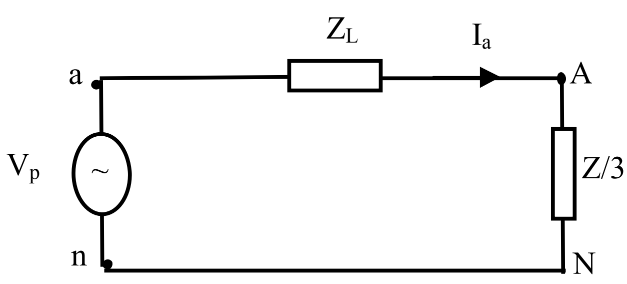
Y-Δ System



The Δ load is converted to its equivalent Y, and the following circuit is obtained.

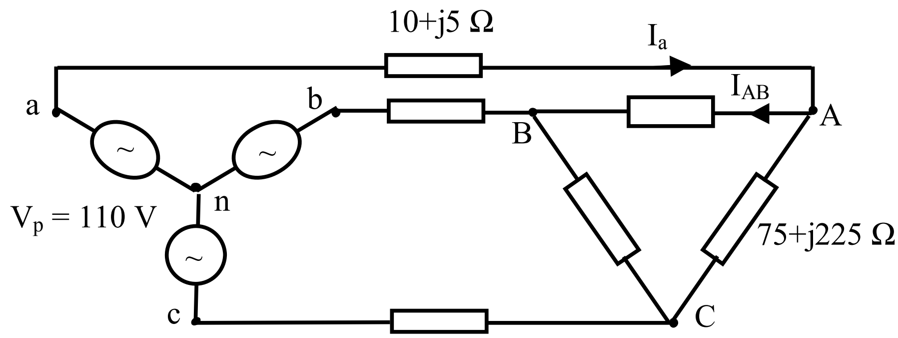


The per phase equivalent circuit is shown below. The above circuit can easily be analyzed as a single phase circuit.

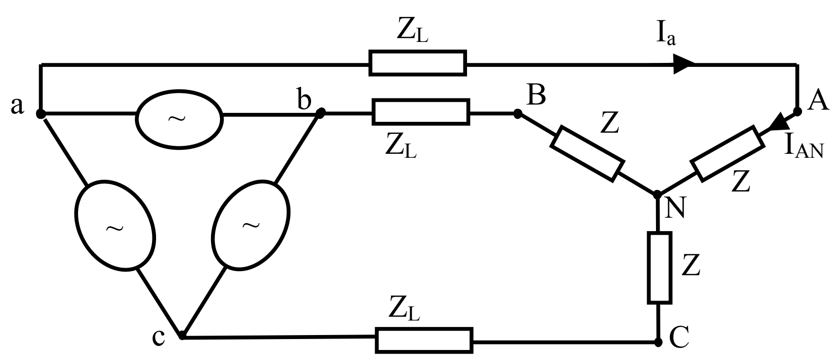


Problem.

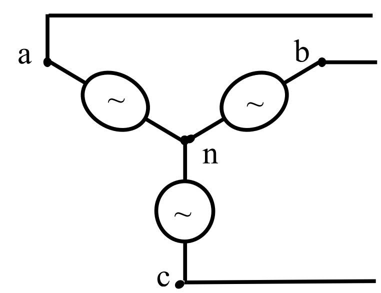
In a balanced Y-Δ 3-ϕ circuit, Y connected source has a V_p = 110V. Line impedance between the source and the load is Z_L = 10 + j5 Ω per phase. Per phase impedance of the Δ-connected load is Z_Δ = 75 + j225 Ω. Determine the phase currents in the Δ-connected load.



Δ-Y System

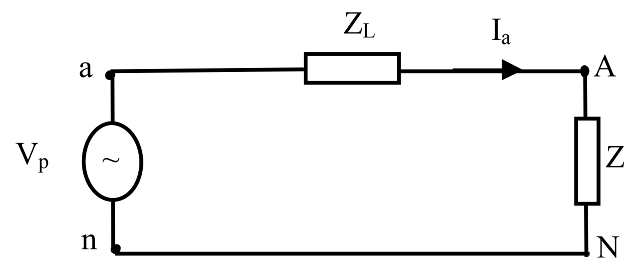


The Δ source is converted to its equivalent Y, as shown by the circuit below.

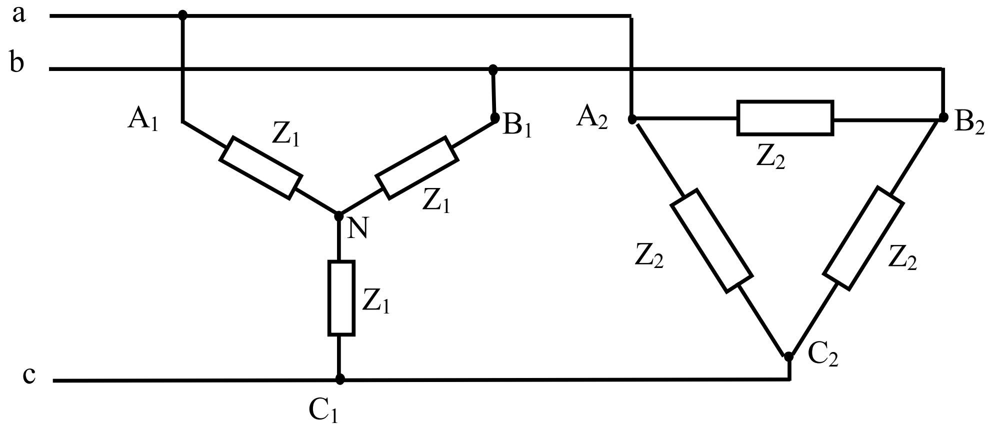


If V_{an} is taken as the reference voltage, then V_{ab} leads V_{an} by 30^0

The per phase equivalent circuit is shown below.



Multiple Balanced Loads in 3-ϕ Systems



A number of balanced 3-ϕ loads are connected in parallel. Such systems can be evaluated using different methods:

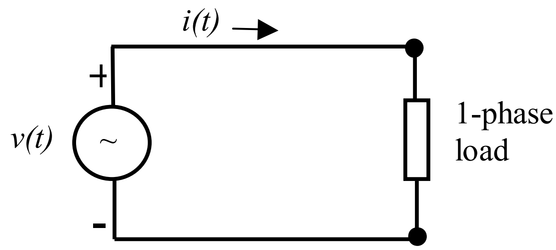
1. Convert all loads to Y, and parallel the impedances
2. Convert all loads to Δ, and parallel the Z's
3. Combine the per phase powers

Problem.

A 3-ϕ motor takes 10 kVA at 0.6 p.f. lagging from a source of 220 volts line-to-line. The motor is in parallel with a balanced Δ load having 16 Ω resistance and 12 Ω capacitive reactance in series in each phase. Find the total VA, P, line current and p.f. of the combined load.

Power Measurement in Power Circuits

Single Phase Power



Instantaneous voltage applied by the source, $v(t) = V_m \sin \omega t$

Instantaneous current drawn by the laod, $i(t) = I_m \sin (\omega t - \theta)$

Assuming the load to be inductive, the current lags the voltage by an angle θ.

The instantaneous power consumed by the load,

$$p(t) = v(t) \,.\, i(t)$$
$$= V_m \sin \omega t \,.\, I_m \sin (\omega t - \theta)$$
$$= \frac{V_m I_m}{2} \cos\theta - \frac{V_m I_m}{2} \cos(2\omega t - \theta)$$

The first term of the instantaneous power equation is a constant value independent of time. This is equivalent to dc power. The second term is a sinusoidal function with twice the excitation frequency. Therefore, single phase power fluctuates with time.

The average power consumed by the single phase load,

$$P = \frac{1}{T}\int_0^T p(t).d(t)$$

$$= \frac{1}{T}\int_0^T \left[\frac{V_m I_m}{2}\cos\theta - \frac{V_m I_m}{2}\cos(2\omega t - \theta)\right] .d(t)$$

$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cos\theta\, d(t) - \frac{1}{T}\int_0^T \left[\frac{V_m I_m}{2}\cos(2\omega t - \theta)\right] d(t)$$

$$= \frac{V_m I_m}{2}\cos\theta\, \frac{1}{T}\int_0^T d(t) - \frac{V_m I_m}{2}\, \frac{1}{T}\int_0^T \cos(2\omega t - \theta)\, d(t)$$

$$= \frac{V_m I_m}{2}\cos\theta - 0$$

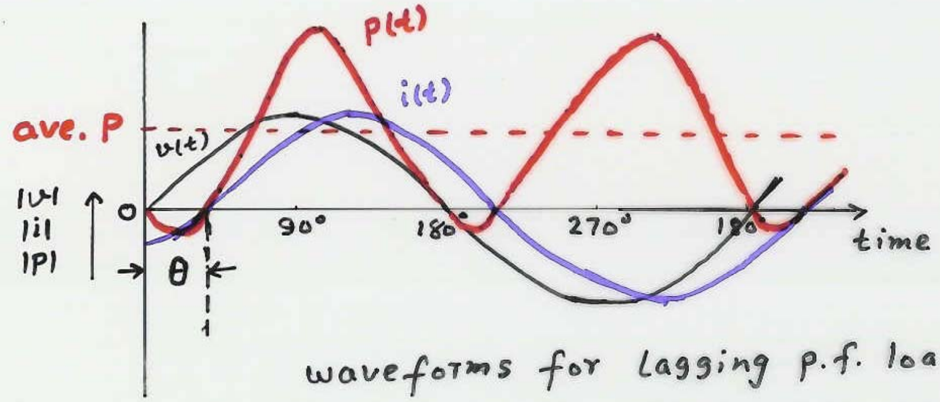
The second term is zero because the average value of a sinusoidal function is zero. Therefore,

$$P = \frac{V_m I_m}{2}\cos\theta$$

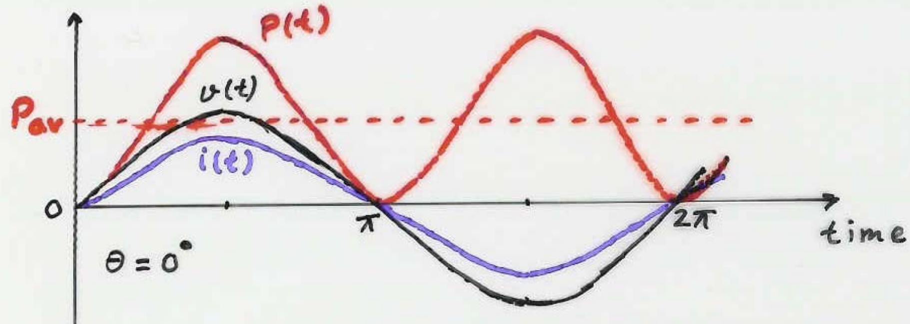
$$= \frac{V_m}{\sqrt{2}}\,\frac{I_m}{\sqrt{2}}\,\cos\theta$$

$$= V.\ I.\ \cos\theta$$ where, V and I are RMS values.

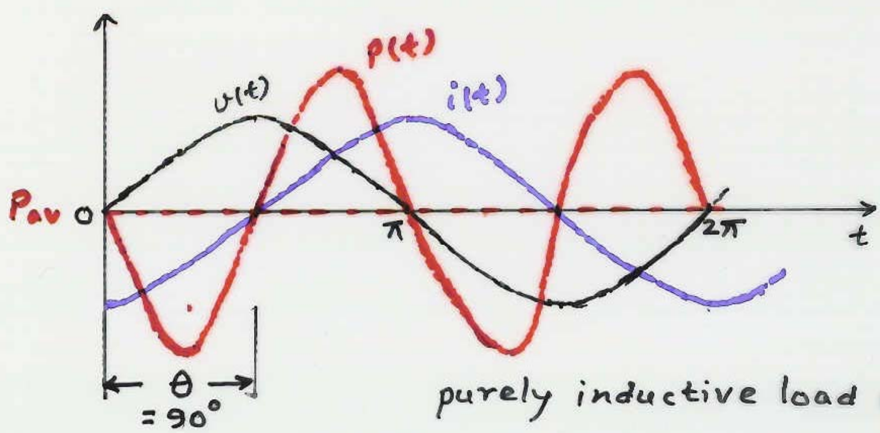
The following figures show the variation of single phase power with time for an inductive load, a resistive load and a purely inductive load respectively. It can be seen that for a purely inductive or capacitive load (i.e. when power factor is zero), the average power (real power) is zero. The power absorbed in one half cycle is released in the other half cycle.



waveforms for lagging p.f. load

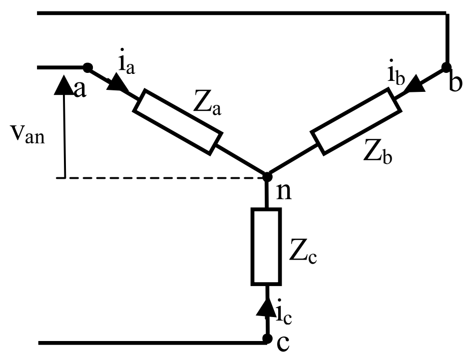


purely resistive load (p.f. = 1.0)



purely inductive load (p.f. = 0 lagging)

Three Phase Power



For phase 'a':

Instantaneous voltage across phase 'a', $v_{an}(t) = V_m \sin \omega t$

Instantaneous current drawn by load, $i_a(t) = I_m \sin (\omega t - \theta)$

Assuming the load is inductive, where the phase current lags the phase voltage by an angle θ.

The instantaneous power consumed by the load in phase 'a',

$$p_a(t) = v_{an}(t) \cdot i_a(t)$$

$$= \frac{V_m I_m}{2}[\cos\theta - \cos(2\omega t - \theta)]$$

Similarly, the instantaneous phase voltage, current and power for phase 'b' are the following considering the normal phase sequence a-b-c.

$$v_{bn}(t) = V_m \sin (\omega t - 120^0)$$

$$i_b(t) = I_m \sin (\omega t - \theta - 120^0)$$

$$p_b(t) = \frac{V_m I_m}{2}[\cos\theta - \cos(2\omega t - \theta + 120^0)]$$

Similarly, for phase 'c':

$$v_{cn}(t) = V_m \sin(\omega t + 120^0)$$

$$i_c(t) = I_m \sin(\omega t - \theta + 120^0)$$

$$p_c(t) = \frac{V_m I_m}{2}[\cos\theta - \cos(2\omega t - \theta - 120^0)]$$

The total instantaneous power in the three phase system is the sum of the powers in the individual phases.

$$p(t) = p_a(t) + p_b(t) + p_c(t)$$

$$= \frac{V_m I_m}{2}[3\cos\theta - \{\cos(2\omega t - \theta) + \cos(2\omega t - \theta + 120^0) + \cos(2\omega t - \theta - 120^0)]$$

Since the term within the curly brackets equals to zero since it is a summation of three equal phasors that are 120^0 out of phase. Therefore,

$$p(t) = \frac{V_m I_m}{2}[3\cos\theta - 0]$$

$$= \frac{3}{2} V_m I_m \cos\theta$$

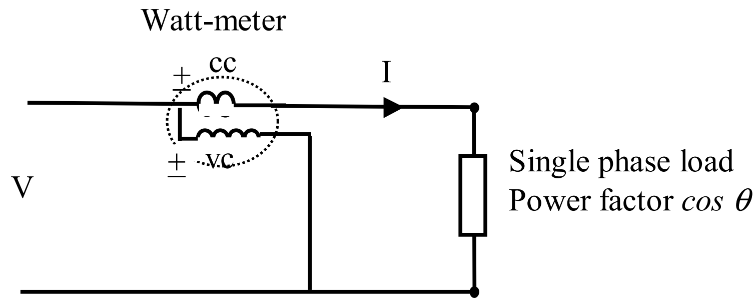
It can be seen from the above expression of three phase power that it does not vary with time, and is always constant. It can be shown that three phase instantaneous power is equal to the three phase average power.

$$p(t) = 3\frac{V_m I_m}{2}\cos\theta = 3\frac{V_m I_m}{\sqrt{2}.\sqrt{2}}\cos\theta$$

$$= 3\, V_{phase\,(rms)}.\, I_{phase\,(rms)}.\, \cos\theta$$

= average three phase power

Watt-meter Connection in Single Phase System



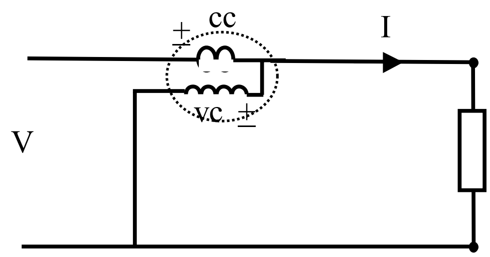
A watt-meter has 4 terminals that must be connected to the circuit in which power is to be measured. Two terminals are for the current coil (CC), and the other two are for the voltage coil (VC). In order to measure the power consumed by a load in a circuit, the 4 terminals should be connected in such a way that the CC carries the current though the load, and the voltage coil has the same voltage as that across the load. The CC has almost zero impedance, so that the voltage drop across it is usually negligible. The VC has infinite impedance, so that it draws nigligible current in normal application. The reference terminals ($\pm$) of both the coils must have the same polarity for a positive torque required for watt-meter measurement.

The power measured by a watt-meter is given by:

$$P_{w\text{-}meter} = \text{voltage across VC x current through CC}$$
$$\text{x cos (angle between VC voltage and CC current)}$$
$$= V\, I \cos\theta \quad \text{(for the above circuit connection)}$$

In the above connection, the VC voltage is not exactly equal to the voltage across the load. It also includes the voltage drop across the CC. The power measurement includes the I^2R power dissipated in the CC. At low loads (relatively low load currents), the voltage drop across the CC is negligible (or the I^2R power in the CC is negligible). The above connection is suitable for low load condition.

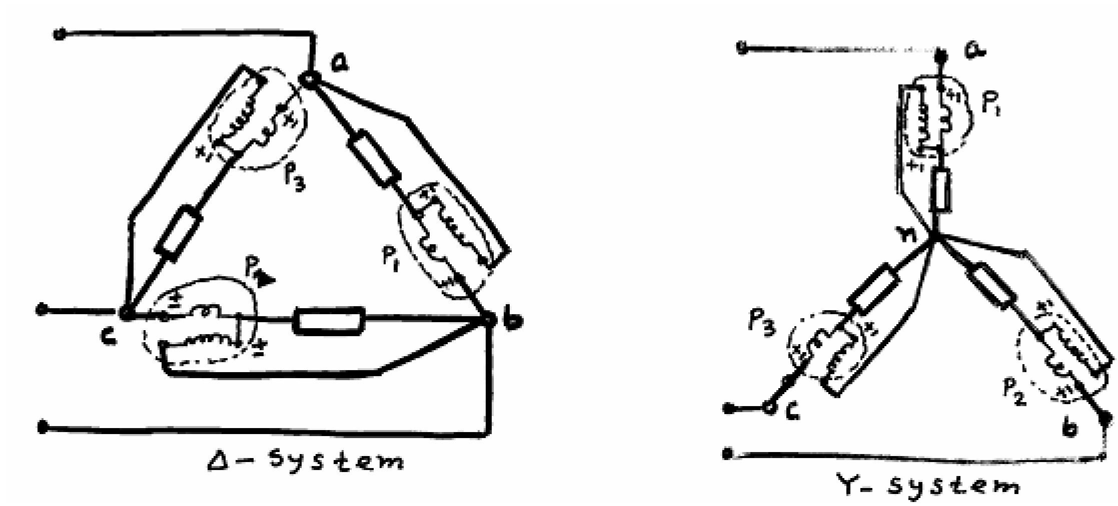
Most watt-meters have CC and VC tap settings such that they can be operated close to the rated values (or at high loads). It should noted that rated values should not be exceeded (over load condition) in a watt-meter appliation. Since watt-meters can often be used in a high load condition by proper selection of the tap settings, the following is a more suitable watt-meter connection in a general appliocation.



In this case, the CC includes the current thorugh the VC. Since the current through the VC is negligible compared to the high load current, the error is insignificant.

Wattmeter Connection in Three Phase System

The total three phase power is the sum of the powers in the three indivudual phases for both balanced and unbalanced systems. A watt-meter can be placed in each phase, and the readings of the three watt-meters summed to obtain the three phase power.



Three phase power = $P_1 + P_2 + P_3$

The above connections are suitable for measurement of three phase power in unbalanced systems. In a balanced three phase system, the power in each phase is equal since each phase is identical. A watt-meter is therefore required in only one phase, and the total power can be calculated by multiplying the watt-meter reading by 3.

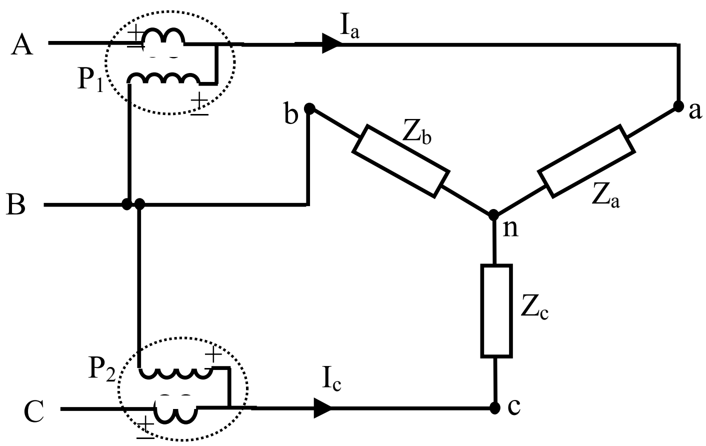
The above Wattmeter connection methods for 3-phase systems are not practical because:

- it is not practical to break the phase of a Δ load
- neutral point in a Y load is not always accessible

Practical methods require wattmeters to be connected across the lines (line-to-line) in 3-ϕ systems. A practical method is known as the ***Two-wattmeter method.***

Two-Wattmeter Method

The figure below shows a balanced three phase load with phase sequence a-b-c.

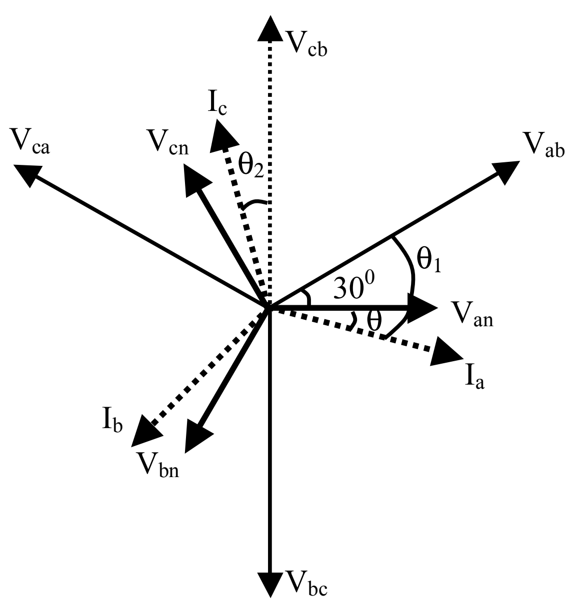


Wattmeter 1 reading, $P_1 = |V_{ab}|.|I_a|. \cos \theta_1$

where θ_1 = angle between V_{ab} and I_a

Wattmeter 2 reading, $P_2 = |V_{cb}|.|I_c|. \cos \theta_2$

where θ_2 = angle between V_{cb} and I_c



p.f. angle, θ

= angle between phase voltage and phase current

= angle between V_{an} and I_a (or V_{cn} and I_c)

θ_1 = angle between V_{ab} and $I_a = 30^0 + \theta$

θ_2 = angle between V_{cb} and $I_c = 30^0 - \theta$

Wattmeter 1 Reading:

$$P_1 = |V_{ab}|.|I_a|.\cos\theta_1 = V_L.I_L.\cos(30^0 + \theta)$$

Wattmeter 2 Reading:

$$P_2 = |V_{cb}|.|I_c|.\cos\theta_2 = V_L.I_L.\cos(30^0 - \theta)$$

Real Power:

$$\begin{aligned} P_2 + P_1 &= V_L.I_L\,[\cos(30^0 + \theta) + \cos(30^0 - \theta)] \\ &= V_L.I_L\,[2\cos 30^0 \cos\theta] \\ &= \sqrt{3}.|V_L|.|I_L|.\cos\theta \\ &= \text{total 3-}\phi\text{ real power} \end{aligned}$$

Reactive Power:

$$\begin{aligned} P_2 - P_1 &= V_L.I_L\,[\cos(30^0 - \theta) - \cos(30^0 + \theta)] \\ &= V_L.I_L\,[2\sin 30^0 \sin\theta] \\ &= |V_L|.|I_L|.\sin\theta \\ \text{i.e.}\quad \sqrt{3}\,(P_2 - P_1) &= \sqrt{3}.\,|V_L|.|I_L|.\sin\theta \\ &= \text{total 3-}\phi\text{ reactive power} \end{aligned}$$

Power Factor Angle:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}V_L I_L \sin\theta}{\sqrt{3}V_L I_L \cos\theta} = \frac{\sqrt{3}(P_2 - P_1)}{P_2 + P_1}$$

$$\text{i.e. } \theta = \tan^{-1}\left[\frac{\sqrt{3}(P_2 - P_1)}{P_2 + P_1}\right]$$

if $P_2 > P_1$ (i.e. $\cos\theta_2 > \cos\theta_1$ or $\theta_2 < \theta_1$), θ is lagging $\rightarrow$ p.f. is lagging

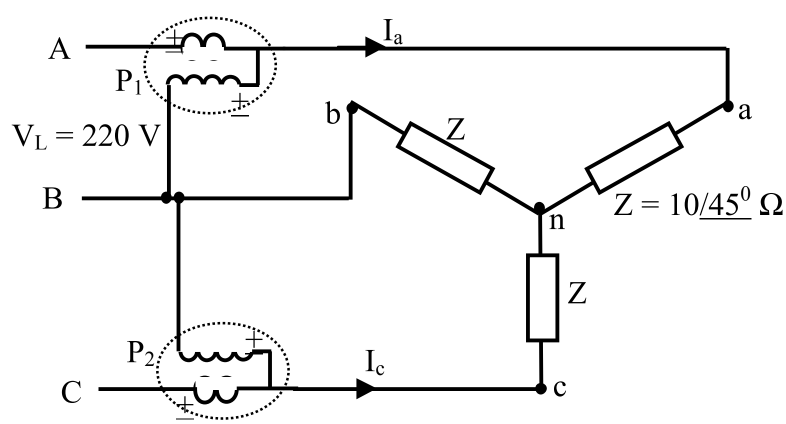
if $P_2 < P_1$, p.f. is leading

Problem.

A balanced Y load having per phase impedance of $10\,\underline{/45^0}\ \Omega$, is supplied by a 3-$\phi$ source with a line-to-line voltage of 220 V. Two-wattmeter method is used to measure the power delivered to the load.

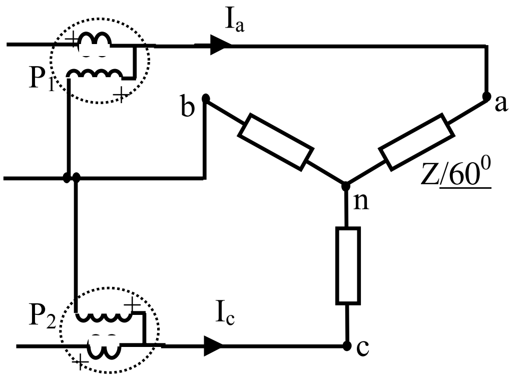
i) Determine the reading on each wattmeter

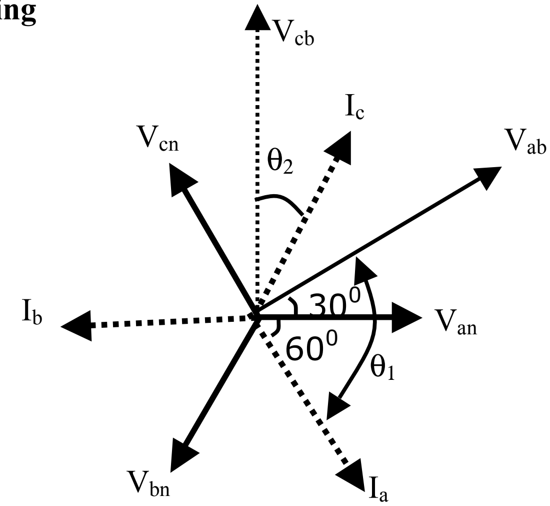
ii) Calculate the total real power, reactive power and the p.f. from the two wattmeter readings.



Two Wattmeter Method: Special Cases

Consider the case when the load p.f. = 0.5 lagging





$P_1 \quad = |V_{ab}|.|I_a|. \cos \theta_1$

$P_2 \quad = |V_{cb}|.|I_c|. \cos \theta_2$

$\theta_1 = 60 + 30 = 90^0$

$\theta_2 = 90 - 60 = 30^0$

$P_1 \quad = V_L I_L \cos 90^0 = 0$

$P_2 \quad = V_L I_L \cos 30^0$

$\quad = (\sqrt{3}/2) V_L I_L = \sqrt{3} V_L I_L \cos 60^0$ = total 3-ϕ power

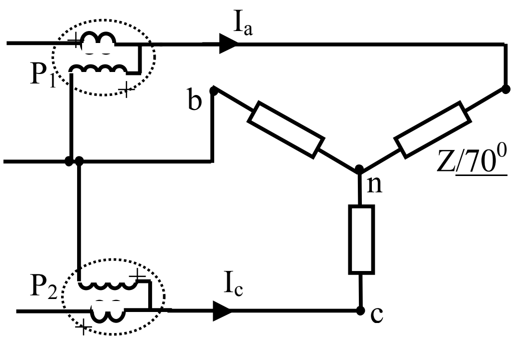
If the load p.f. = 0.5 leading,

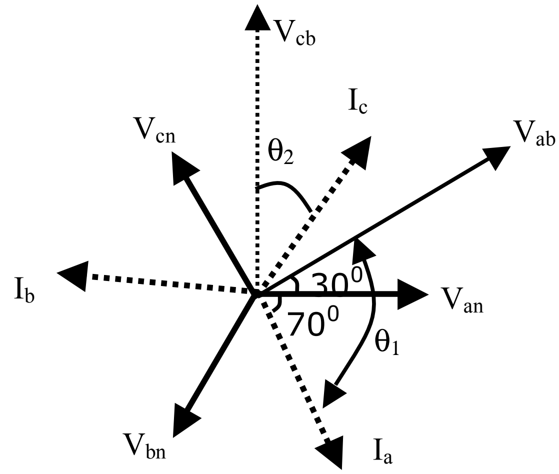
$P_1 \quad$ = total 3-ϕ power

$P_2 \quad = 0$

Consider the case when the load p.f. < 0.5 lagging

Power factor angle, $\theta > 60^0$





$P_1 \quad = |V_{ab}|.|I_a|. \cos \theta_1$

$P_2 \quad = |V_{cb}|.|I_c|. \cos \theta_2$

Since $\theta_1 > 90^0$, a negative torque is produced in Wattmeter 1.

The voltage coil connections must be reversed in Wattmeter 1 to obtain a forward (upward) reading.

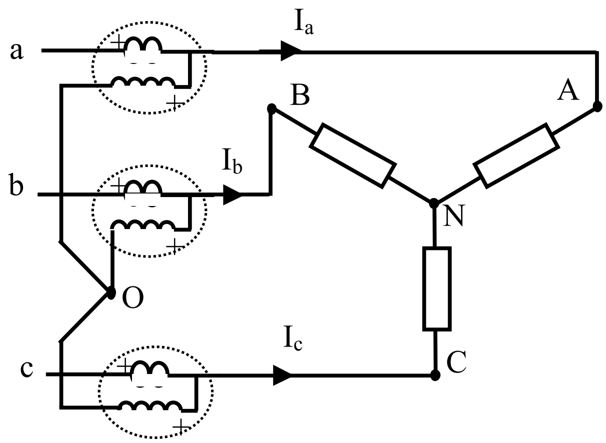
Then, Total Power = P_2 - P_1

The reading in the wattmeter with reversed VC connection should be reckoned as negative.

If p.f. < 0.5 leading, the voltage coil in Wattmeter 2 must be reversed.

Power Measurement in 3-ϕ Unbalanced Systems

Two Wattmeter method is also applicable to unbalanced 3-ϕ, 3-wire systems.



$$v_{AO} = v_{AN} + v_{NO}$$

$$v_{BO} = v_{BN} + v_{NO}$$

$$v_{CO} = v_{CN} + v_{NO}$$

Average power delivered to the 3-ϕ load,

$$P_{abc} = \frac{1}{T}\int_0^T [p_a(t) + p_b(t) + p_c(t)]\,dt$$

$$= \frac{1}{T}\int_0^T [v_{AN}\, i_a + v_{BN}\, i_b + v_{CN}\, i_c]\,dt$$

The total average power measured by the 3 wattmeters,

$$P_{meters} = P_1 + P_2 + P_3 = \frac{1}{T}\int_0^T [v_{AO}\, i_a + v_{BO}\, i_b + v_{CO}\, i_c]\,dt$$

$$= \frac{1}{T}\int_0^T \left[(v_{AN}\, i_a + v_{BN}\, i_b + v_{CN}\, i_c) + v_{AN}\,(i_a + i_b + i_c)\right] dt$$

But since $(i_a + i_b + i_c) = 0$,

$$P_{meters} = \frac{1}{T}\int_0^T \left[v_{AN}\, i_a + v_{BN}\, i_b + v_{CN}\, i_c\right] dt \quad = P_{abc}$$

The 3 wattmeters measure the total power irrespective of the potential of the point “O”.

If the point “O” were placed on the line “b”, wattmeter on line “b” will read zero (voltage across VC = 0). The sum of the other two wattmeter readings gives the total power.

Therefore, two-wattmeter method can be used to measure power in unbalanced 3-ϕ, 3-wire systems.

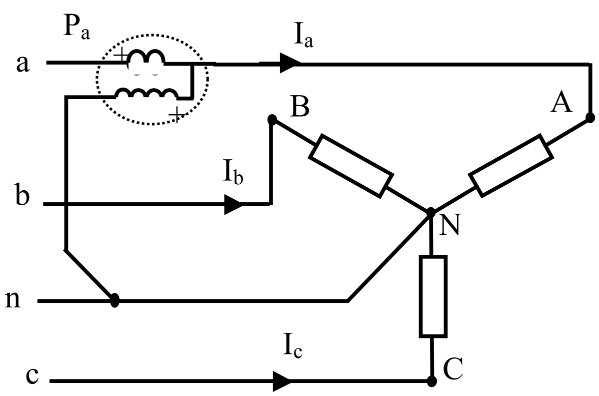
Power Measurement in 3-ϕ, 4-wire systems

Balanced 3-ϕ, 4-wire systems

In balanced 3-ϕ, 4-wire systems, two-wattmeter method can be used since $(i_a + i_b + i_c) = i_N = 0$.

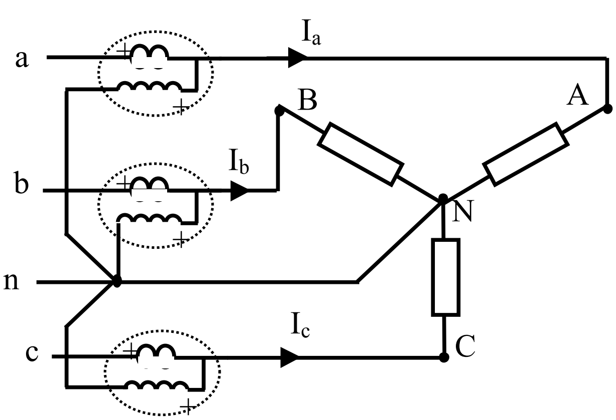
Only one wattmeter is necessary since

- the power in each phase is equal
- the neutral point is accessible for wattmeter connection



Total Power $= 3 \times P_a$

Unbalanced 3-ϕ, 4-wire systems

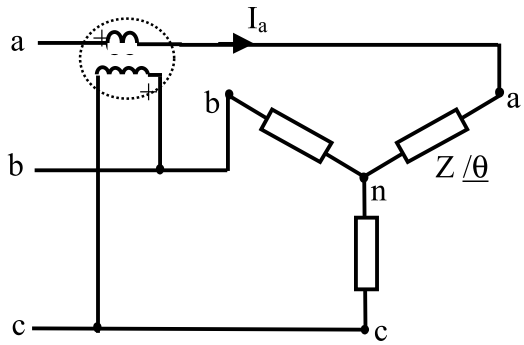


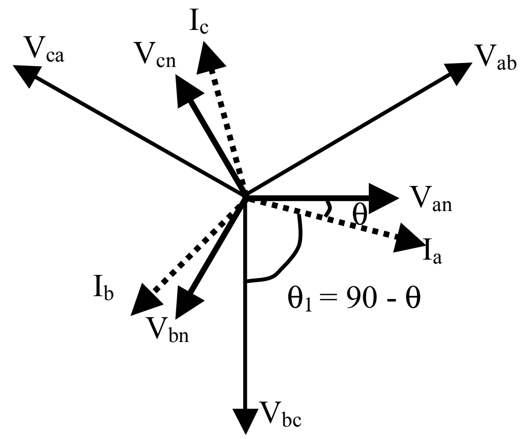
In unbalanced 3-ϕ, 4-wire systems, two-wattmeter method cannot be used since $(i_a + i_b + i_c) \neq 0$. Therefore, three wattmeters are required across the lines.

$$\text{Total Power} = P_a + P_b + P_c$$

Reactive Power Measurement in a Balanced 3-ϕ System

A single wattmeter can be used to measure the reactive power in a balanced 3-ϕ system.





Wattmeter reading,

$$P = |V_{bc}|.|I_a|.\cos\theta_1$$

where θ_1 = angle between V_{bc} and I_a

$$= 90^0 - \theta$$

i.e. $$P = |V_L|.|I_L|.\cos(90^0 - \theta)$$

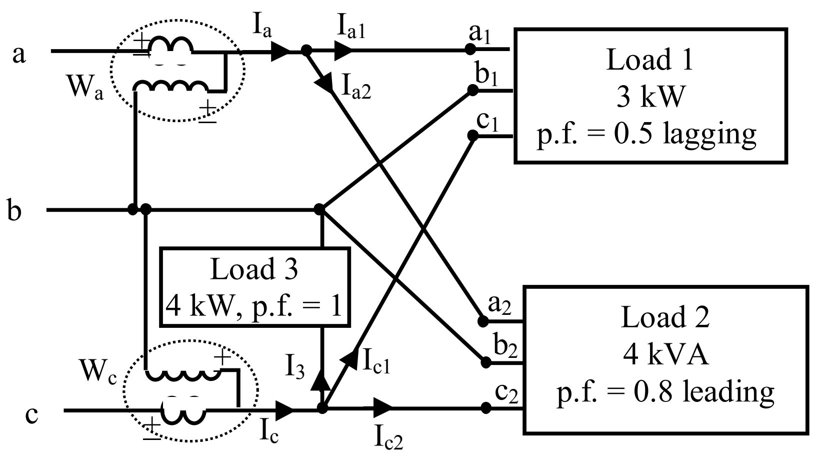
$$\sqrt{3}\,P = \sqrt{3}.\,|V_L|.|I_L|.\sin\theta = \text{total 3-}\phi\text{ reactive power}$$

If the load power factor is leading, $\theta_1 > 90^0$, and therefore, the voltage coil connection must be reversed. Wattmeter reading is considered negative.

Total Q = - √3 x Wattmeter reading (i.e. Q generated)

Problem.

Balanced 3-ϕ voltages, with a-b-c phase sequence and 200 V line voltage, are applied to two balanced 3-ϕ loads and one 1-ϕ load across lines "b" and "c". The balanced 3-ϕ loads take 3 kW at 0.5 p.f. lagging and 4 kVA at 0.8 p.f. leading respectively. The 1-ϕ load takes 4 kW at unity p.f. Two Wattmeters W_a and W_c have their CC in lines "a" and "c" respectively, and their common VC connection on line "b". What are the readings on the two wattmeters?



Per Unit (p.u.) System

Computations related to power systems involving machines and transformers are often carried out in p.u. system.

In p.u. system, power system quantities (such as V, I, R, X, VA, P and Q) are expressed as decimal fractions of chosen base values.

$$\text{p.u. value} = \frac{\text{actual value}}{\text{base value}}$$

Advantages of p.u. system

p.u. values of circuit parameters are the same on either side a transformer. We don't have to worry about circuit parameters being referred to one side or the other.

Machines and transformer constants (e.g. resistance, leakage/transient/sub-transient reactance, excitation current, etc.) lie within a narrow range of values when expressed in p.u. We can therefore quickly estimate their correctness.

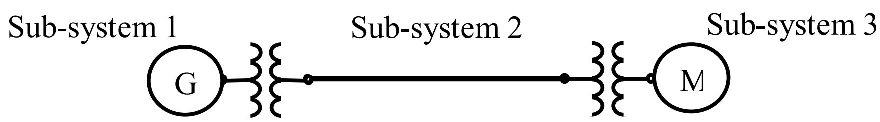
e.g. Leakage reactance of autotransformers lie within 0.01 – 0.04 p.u. (or 1% - 4%).

Procedure for applying p.u. system

1. Choose base values for 2 quantities, the VA and the V. Other base quantities (i.e. I_{base} and Z_{base}) are determined from the chosen VA_{base} and V_{base}.

2. Choose one VA_{base} for the entire system under study. Usually VA_{base} is the rated VA of the transformer or generator in the system.

3. Divide the system into sub-systems whenever a transformer is encountered. Choose a different V_{base} for each sub-system. The ratio of the V_{base} on each side of a transformer must be equal to the transformation ratio.



Power System Under Study

4. Determine I_{base} and Z_{base} for each sub-system.

$$I_{base} = \frac{VA_{base}}{V_{base}} \qquad Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^{2}}{VA_{base}}$$

5. Calculate the p.u. values of the system quantities.

$$\text{p.u. value} = \frac{\text{actual value}}{\text{base value}}$$

(Use Z_{base} for R, X and Z. Use VA_{base} for VA, P and Q)

6. Carry our circuit analysis with computations in p.u.

7. Obtain the actual values of the desired quantities using the equation:

Actual value = p.u. value x Base value

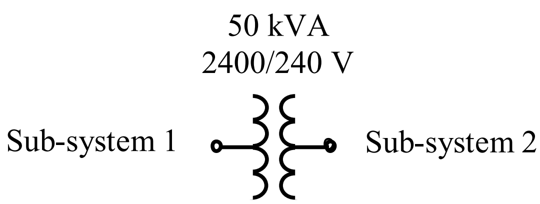
Note:

p.u. values are often expressed in percentage.

Power equipment impedance ratings are usually given in p.u. (expressed in %) based on their rated voltage and VA.

Example.

A 50 kVA, 2400/240 V transformer has a leakage impedance of 1.0 + j3.61 Ω referred to HV side. Show that the p.u. values are the same on either side of the transformer.



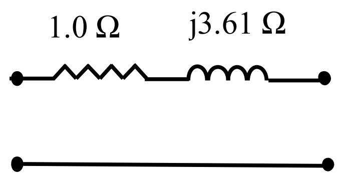
Solution:

For entire system, $VA_{base} = 50{,}000$ VA

HV side (sub-system 1):	LV side (sub-system 2):
$V_{base} = 2400$ V	$V_{base} = 240$ V
$Z_{base} = \dfrac{V_{base}^{\;2}}{VA_{base}} = \dfrac{2400^2}{50000} = 115.2\ \Omega$	$Z_{base} = \dfrac{240^2}{50000} = 1.152\ \Omega$
$Z_{actual} = 1.0 + j3.61\ \Omega$	Z (referred to LV side) $= \dfrac{Z_{HV}}{a^2}$ $= \dfrac{1.0 + j3.61}{10^2} = 0.01 + j0.0361\ \Omega$
$Z_{p.u.} = \dfrac{Z_{actual}}{Z_{base}}$ $= \dfrac{1.0 + j3.61}{115.2} = 0.00868 + j0.0314$ p.u.	$Z_{p.u.} = \dfrac{0.01 + j0.0361}{1.152} = 0.00868 + j0.0314$ p.u.

Or $R_{p.u.}$ = 0.868% and $X_{p.u.}$ = j3.14%

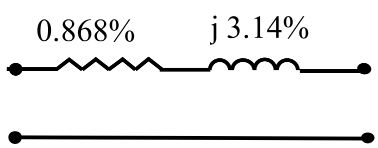
Transformer equivalent circuit using actual values must either be referred to the HV side or to the LV side.



referred to HV side

referred to LV side

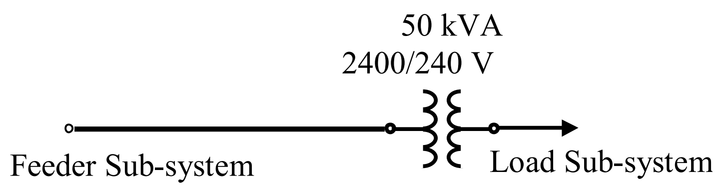
Since p.u. values are the same on either side of a transformer, its equivalent circuit using p.u. values is the same when viewed from either side:



equivalent circuit

Problem.

A 50 kVA, 2400/240 V, 60 Hz transformer, with a leakage impedance of 1.42+j1.82 Ω on the HV side, is used to step down the voltage at the load end of a feeder whose impedance is 0.3+j1.6 Ω. The voltage at the sending end of the feeder is 2400 V. Find the voltage at the secondary terminals of the transformer when the connected load draws rated current at a p.f. of 0.8 lagging.



Problem.

A 50 kVA, 2400/240 V transformer has a leakage impedance of 1.0 + j3.61 Ω referred to HV side. Calculate the voltage regulation at 0.85 p.f. lagging.

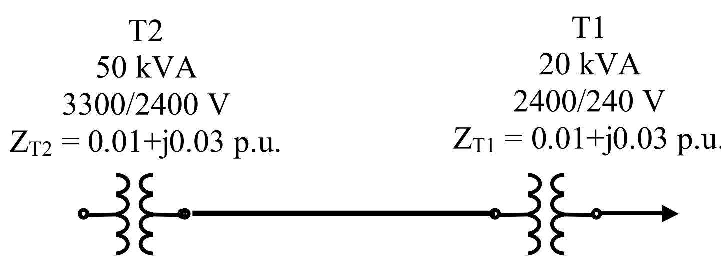
Change of Base

Power equipment impedance ratings are usually given in p.u. (expressed in %) based on their rated voltage and VA.

When chosen bases values are different from the equipment rated values, the rated impedance (in p.u.) of the equipment must be converted to the chosen base values.

$$Z_{p.u.\,(new)} = Z_{p.u.\,(old)} \times \left(\frac{V_{base\,(old)}}{V_{base\,(new)}}\right)^2 \times \frac{VA_{base\,(new)}}{VA_{base\,(old)}}$$

Example:



VA_{base} (for the entire system) = 50 kVA (change for T1)

V_{base} taken same as equipment ratings (no change)

Z_{T1} on the new base,

$$Z_{T1(new)} = Z_{p.u.\,(old)} \times \frac{VA_{base\,(new)}}{VA_{base\,(old)}}$$

$$= (0.01+j0.03) \times \frac{50}{20} = 0.025 + j0.075 \text{ p.u.}$$