CHAPTER 3: OSCILLATORS AND WAVEFORM-SHAPING CIRCUITS

In the design of electronic systems, the need frequently arises for signals having prescribed standard waveforms (e.g., sinusoidal, square, triangle, pulse, etc). These waveforms are commonly used in computers, control systems, communication systems and test measurement systems.

There are two common ways for generating sinusoids:
1. Positive feedback loop with non-linear gain limiting
2. Appropriately shaping other waveforms such as a triangle waves.

Circuits that directly generate square, triangle and pulse waveforms generally employ circuit blocks known as multivibrators. Three basic types are bistable, astable and monostable.

I. SINUSOIDAL OSCILLATORS:

Commonly referred to as linear sine-wave oscillators although some forms of non-linearity have to be employed to limit the output amplitude. Analysis of the circuits is more difficult as s-plane analysis cannot be directly applied to the non-linear part of the circuit. The basic structure of a sinusoidal oscillator consists of an amplifier and a frequency selective network connected in a positive feedback loop.

\[
\begin{align*}
A_f(s) &= \frac{A(s)}{1-A(s)\beta(s)} \\
L(s) &= A(s)\beta(s) \\
1 - L(s) &= 0
\end{align*}
\]

If at a specific frequency \(f_0\), the loop gain \(A\beta\) is equal to unity, it follows that \(A_f\) will be infinite. Such a circuit is by definition an oscillator.
Thus for the sinusoidal oscillator at $\omega_0$:

$$L(j\omega_0) = A(j\omega_0) \cdot \beta(j\omega_0) = 1$$

This condition is called Barkhausen Criteria for oscillation, in which:

"UNITY GAIN, ZERO PHASE SHIFT"

It should be noted that the frequency of oscillation $\omega_0$ is determined by the phase characteristics of the feedback loop. The loop oscillates at the frequency for which the phase is ZERO. The steeper the phase shift as a function of frequency $\phi(\omega)$, the more stable the frequency of oscillation.

---

**NON-LINEAR AMPLITUDE CONTROL:**

Generally, it is difficult to design circuits with $A\beta=1$ as circuit parameters vary with temperature, time, and component values.

- If $A\beta < 1$ oscillator ceases,
- If $A\beta > 1$ oscillation grows until circuit saturates.

It is required to have a mechanism to force $A\beta=1$. This is accomplished by employing a non-linear circuit for gain control:

- Design circuit with $A\beta > 1$ as voltage of oscillation increases, gain control mechanism kicks in and reduces gain to 1.
- Design circuit with right half plane poles. The gain control pulls the poles back to the imaginary axis.

**Two approaches:**
1. *The first approach* uses a limiter circuit, oscillations are allowed to grow until the level reaches the limiter set value. Once the limiter comes into operation, the amplitude remains constant. The limiter should be designed to minimize non-linear distortion.

2. *The second method* uses a resistive element in the feedback loop whose resistance can be controlled by the sinusoidal output amplitude. Diodes or JFETs (operating in triode region) are commonly used.

A popular limiter circuit for amplitude control can be seen below:
II. OPAMP - RC OSCILLATORS:

1. WIEN-BRIDGE OSCILLATOR

The loop gain can be found by multiplying the transfer function of the feedback path, \( V_a(s)/V_0(s) \), by the amplifier gain.

\[
L(s) = \left[1 + \frac{R_2}{R_1}\right] \frac{Z_P}{Z_P + Z_S}
\]

\[
L(j\omega) = \frac{1 + \frac{R_1}{R_2}}{3 + j(\omega RC - \frac{1}{\omega RC})}
\]

The loop gain will be a real number (i.e., the phase will be zero) at one frequency \( \omega_0 \) given by:

\[
\omega_0 RC = \frac{1}{\omega_0 RC}
\]

\[
\omega_0 = \frac{1}{RC}
\]

To obtain sustained oscillation at this frequency, the magnitude of the loop gain should be unity which can be achieved by setting:

\[
\frac{R_2}{R_1} = 2
\]

To ensure that oscillation starts, one chooses \( R2/R1 \) slightly greater than 2.
The amplitude of the oscillation can be controlled using a non-linear limiter as seen below.

2. PHASE SHIFT OSCILLATOR

Figure 12.7 shows the basic structure of the phase shift oscillator. It consists of a negative gain amplifier (-K) with a three-section (3\textsuperscript{rd} order) RC ladder network in the feedback.

The circuit will oscillate at the frequency for which the phase shift of the RC network is 180°. Only at this frequency will the phase shift around the loop be 0° (360°). Three RC sections are required to produce a 180° phase shift at a finite frequency.

The value of K is chosen to be slightly higher than the inverse of the magnitude of the RC network transfer function at the frequency of oscillation.
3. **ACTIVE FILTER TUNED OSCILLATOR**

In this type of filter, Figure 12-10, a filter is used to select a particular frequency in the spectrum of a square wave (usually the fundamental frequency). Output of the filter is a sinewave and is taken as the output of the oscillator. The output is fed back to a limiter which is used to convert a sinewave to a squarewave. The squarewave signal then becomes the input of the filter.

The actual circuit is shown in Figure 10.11, the limiter is a pair of diodes to have a squarewave at $v_2$. This filter is an active filter (we will study this filter later) to select the fundamental frequency and provides the output at $v_1$.

The op-amp RC oscillator circuits are useful for operation in the 10Hz-1MHz range due to limitations in passive component size (low frequency) and op-amp slew rate (high frequency). For higher frequencies, circuits that employ transistors together with LC tuned circuits or crystals are commonly used.
III. LC AND CRYSTAL OSCILLATORS:

Oscillators utilizing transistors and LC tuned circuits or crystals are useful for operation in the range from 100KHz to 500MHz. They exhibit higher Q than RC types (more stable). However, LC oscillators are difficult to tune over wide range of frequency and crystal oscillator operates at a single frequency. The extremely stable response of the crystal oscillators has made them very popular, particularly for digital timing signals.

**LC TUNED OSCILLATOR**

Two common used configurations are the Colpitts and the Hartley oscillators. The basic circuit structures without biasing can be seen below.
Both circuits utilize a parallel LC circuit connected between the collector and the base with a fraction of the tuned circuit voltage fed to the emitter of the transistor. The resistor R models the losses of the inductor, the load resistance of the oscillator and the output resistance of the transistor.

If the frequency of operation is sufficiently low, we can neglect the transistor parasitic capacitances. The frequency of oscillation is determined by the resonant frequency of the parallel tuned circuit (also known as a tank circuit). For the Colpitts oscillator:

\[ \omega_0 = \frac{1}{\sqrt{L \left( \frac{C_1 C_2}{C_1 + C_2} \right)}} \]

For the Hartley oscillator

\[ \omega_0 = \frac{1}{\sqrt{C (L_1 + L_2)}} \]

The ratio \( L_1/L_2 \) or \( C_1/C_2 \) determines the feedback factor and thus must be adjusted in conjunction with the transistor gain to ensure that oscillations will start.

To determine the oscillation condition for the Colpitts oscillator, we replace the transistor with its equivalent circuit. To simplify the analysis, we neglect the transistor capacitances except capacitance \( C_{BE} \) is a part of \( C_2 \).
A node equation at the transistor collector (C) yields:

\[ sC_2 V_\pi + g_m V_\pi + \left(\frac{1}{R} + sC_1\right)(1 + s^2LC_2)V_\pi = 0 \]

Since \( V_\pi \neq 0 \) (oscillations have started), it can be eliminated (i.e., the other terms are zero).

\[ s^3LC_1C_2 + s^2\left(L\frac{C_2}{R}\right) + s(C_1 + C_2) + \left(g_m + \frac{1}{R}\right) = 0 \]

\[ \left(g_m + \frac{1}{R} - \frac{\omega^2LC_2}{R}\right) + \omega(C_1 + C_2) - \omega^3LC_1C_2 = 0 \]

**For oscillations to start, both the real and imaginary parts must be zero.**

Setting the imaginary part to zero gives

\[ \omega_0 = \frac{1}{\sqrt{\frac{C_1}{L\left(C_1 + C_2\right)}}} \]

which is the resonant frequency of the tank circuit.

Setting the real part to zero yields

\[ \frac{C_2}{C_1} = g_mR \]

For sustained oscillation, the magnitude of the gain from the base to collector (\( g_mR \)) must be equal to the inverse of the voltage ratio provided by the capacitive divider:

\[ \frac{v_{be}}{v_{ce}} = \frac{C_1}{C_2} \]

For oscillation to start, the loop gain must be greater than unity which is equivalent to

\[ g_mR > \frac{C_2}{C_1} \]

As oscillation grows in amplitude, the transistors non-linear characteristics reduce the loop gain to unity, thus sustaining oscillations. An example of a complete Colpitts oscillator is shown below
The radio frequency choke (RFC) in this oscillator provides a high reactance at \( \omega_0 \) but a low DC resistance. Unlike the op-amp oscillators that incorporate special amplitude control circuitry, LC tuned oscillators utilize the non-linear \( i_c-v_{be} \) characteristics of the BJT (or \( i_d \) versus \( v_{gs} \) for FET) for amplitude control. As the oscillations grow, the effective gain of the transistor is reduced below its small signal value. The LC tuned oscillators are known as self-limiting oscillators.

Reliance on the non-linear characteristics of the BJT (or the FET) implies that the collector (drain) current waveform will be nonlinearity distorted. Nevertheless, sinusoidal of high purity because of the filtering action of the LC tuned circuit.

**CRYSTAL OSCILLATORS**

A piezoelectric crystal, such as quartz, exhibits electro-mechanical resonant characteristics that are very stable (with time and temperature) and high selectivity (having very high Q factor). The circuit symbol of a crystal is shown below.

The resonant properties are characterized by a large inductance \( L \) (as high as hundreds of Henrys), a very small series capacitance \( C_s \) (as small as 0.0005pF), a series resistance \( r \) representing a Q factor \( Q=\omega_0 L/r \) that can be as high as few hundred thousand) and a parallel capacitance \( C_p \) (a few picoFarad).
Capacitance $C_p$ represents the electrostatic capacitance between the two parallel plates of the crystal ($C_p >> C_s$). Since the Q factor is so high, we can neglect the resistance $r$ and express the crystal impedance as:

$$ Z(s) = \frac{1}{sC_p + \frac{1}{sL + 1/sC_s}} $$

which can be manipulated to the form

$$ Z(s) = \frac{1}{sC_p s^2 + \frac{1}{LC_s}} $$

we see that the crystal has two resonant frequencies:

$$ \omega_s = \frac{1}{\sqrt{LC_s}} \quad \text{and} \quad \omega_p = \frac{1}{\sqrt{L \frac{C_p C_s}{C_s + C_p}}} $$

they are **series resonance and parallel resonance**.

For $s=j\omega$

$$ Z(j\omega) = -j \frac{1}{\omega C_p} \left( \frac{\omega^2 - \omega^2_s}{\omega^2 - \omega^2_p} \right) $$

It can be seen that $\omega_p > \omega_s$, however, since $C_p >> C_s$, the two resonant frequencies are very close.
From Figure 12.15, we observe that the crystal reactance is inductive over a narrow frequency band between $\omega_p$ and $\omega_s$. We may use the crystal to replace the inductor in a Colpitts oscillator. The resulting circuit will oscillate at the resonant frequency of the crystal inductance $L$ with the series equivalent of $C_s$ and $(C_p + \frac{C_1C_2}{C_1 + C_2})$.

Since $C_s$ is much smaller than the other capacitances, it will dominate and

$$\omega_0 = \frac{1}{\sqrt{LC_s}} = \omega_s$$

A popular configuration of the Colpitts oscillator called Pierce oscillator is shown below.

Resistor $R_f$ determines the DC operating point in the high gain region of the CMOS inverter. Resistor $R_1$ together with capacitor $C_1$ provides a LPF that discourages the circuit from oscillating at higher harmonic of the crystal frequency.

Common to purchase crystal modules with TTL, CMOS or ECL outputs with 14 pin dip or surface mount. Crystals are available in standard frequencies and can be custom ordered for relatively low cost. The oscillators can be tuned a small amount with the use of variable capacitor (varactor) to create Voltage Control Crystal Oscillator (VCXO). Crystals are also be used in high Q filters such as crystal filters or SAW filters.
VOLTAGE CONTROLLED OSCILLATORS (VCO)

There are IC oscillators with output rate variable over some range of frequency according to an input control voltage. Some have frequency range from 1000:1. An example is 74LS624, the IC generates digital logic levels up to 20MHz using external RC to set. Faster VCO available in the 200MHz-1GHz range. Many VCO use external crystals for accuracy. Commonly varactor is used to control frequency of oscillation.

IV. MULTIVIBRATORS:

1. Bistable Multivibrator:

This multivibrator has two stable states. The circuit can remain in either stable state indefinitely and moves to the other stable state only when triggered. Bistability can be obtained by connecting an amplifier in a positive feedback loop having loop gain greater than unity.

This circuit has 2 stable states, one with the op-amp in positive saturation and the other with the op-amp in negative saturation.
**Triggering the bistable circuit:**

If the circuit is in the positive saturation (L+) state, it can be switched to the negative saturation (L-) state by applying an input \( v_I \) of value greater than \( V_{TH} = \beta L_+ \).

\( v_I \) initiates or triggers regeneration. Thus we can remove \( v_I \) with no effect on the regeneration process, \( v_I \) can simply be a pulse which is commonly referred to as a trigger signal. The circuit is known as the Schmitt trigger.

A simple change in the input converts the circuit into a non-inverting bistable circuit.
The output levels of the bistable circuit adjusted by cascading the op-amp with a limiter circuit.

2. Astable Multivibrator:

A square waveform can be generated by making a bistable multivibrator switch state periodically. This can be done by connecting the bistable multivibrator with an RC circuit in the feedback loop.

This circuit has no stable state and is called an astable multivibrator.
The voltage at the op-amp inverting terminal during charging cycle is:

\[ v_\text{-} = L_+ - (L_+ - \beta L_-)e^{-\frac{t}{\tau}} \quad \text{where} \quad \tau = RC \]

Substituting \( v_\text{-} = \beta L_+ \) at \( t=T_1 \) gives

\[ T_1 = \tau \ln\left(\frac{1 - \beta L_-}{L_+ - \beta L_-}\right) \]

Similarly for the discharge cycle, it can be shown

\[ T_2 = \tau \ln\left(\frac{1 - \beta L_+}{L_- - \beta L_-}\right) \]

If \( L_+ = L_- \) and \( T = T_1 + T_2 \) then

\[ T = 2\tau \ln\left(\frac{1 + \beta}{1 - \beta}\right) \]

- The square wave generator can be made to have variable frequency by adjusting \( C \) and/or \( R \).
- The waveform across \( C \) can be made almost triangular by using a small value for the parameter \( \beta \).
Generation of triangle waveforms:

The exponential waveform generated in the astable circuit can be changed to triangular by replacing the low pass RC circuit with an integrator. The integrator causes linear charging and discharging of the capacitor. Because the integrator is inverting, it is necessary to use the non-inverting bistable circuit.

![Bistable Circuit Diagram](image)

**Fig. 12.25** General scheme for generating triangular and square waveforms.

\[
\frac{V_{TH} - V_{TL}}{T_1} = \frac{L_+}{CR} \quad T_1 = CR \frac{V_{TH} - V_{TL}}{L_+} \\
\frac{V_{TH} - V_{TL}}{T_1} = -\frac{L_-}{CR} \quad T_2 = CR \frac{V_{TH} - V_{TL}}{-L_-}
\]

IF \(L_+ = L\), then symmetrical waveforms are obtained

3. **Monostable Multivibrator:**

In some applications, the need arises for a pulse of known height and width generated in response to a trigger signal. Because the width of the pulse is predictable, its trailing edge can be used for timing purposes. Such a pulse can be generated by a monostable multivibrator.

The monostable multivibrator has one stable state in which it can remain indefinitely. It also has a quasi-stable state in which it remains for a predetermined interval equal to the desired width of the output pulse. Once the interval expires, the monostable returns to the stable state and remains there awaiting another triggering signal. The circuit is commonly called a one shot.

The monostable circuit shown below is an augmented form of the astable circuit.
The duration $T$ of the output pulse is determined by the exponential waveform at $v_B$.

$$v_B(t) = L_\bot - (L_\bot - V_{D1})e^{-\frac{t}{C_1R_3}}$$

By substituting $v_B(T) = \beta L_\bot$,

$$\beta L_\bot = L_\bot - (L_\bot - V_{D1})e^{-\frac{T}{C_1R_3}}$$

$$T = C_1R_3 \ln\left(\frac{V_{D1} - L_\bot}{\beta L_\bot - L_\bot}\right)$$

For $V_{D1} << |L|$, this equation can be approximated by

$$T \approx C_1R_3 \ln\left(\frac{1}{1 - \beta}\right)$$

**Note:** the monostable should not be re-triggered again until $C1$ has been recharged to $V_{D1}$ (*recovery period*)
INTEGRATED CIRCUIT TIMERS

Commercially available integrated circuit packages contain a bulk of the circuitry needed to implement monostable and astable multivibrators having precise characteristics. The most popular of such IC’s is the 555 timer.

Fig. 12.27 Block diagram representation of the internal circuit of the 555 integrated-circuit timer.
Monostable circuit using 555 timer

\[ v_C = V_{CC} \left( 1 - e^{-\frac{t}{RC}} \right) \quad v_C = V_{TH} = \frac{2}{3} V_{CC} \quad t = T \Rightarrow \quad T = CR \ln(3) = 1.1CR \]
Astable circuit using 555 timer:

Fig. 12.29 (a) The 555 timer connected to implement an astable multivibrator. (b) Waveforms of the circuit in (a).
The rise in $v_C$ is given by:

$$v_C = V_{CC} - (V_{CC} - V_{TL})e^{-\frac{t}{C(R_A + R_B)}}$$

$$v_C = V_{TH} = \frac{2}{3}V_{CC} \quad t = T_H \Rightarrow \quad V_{TL} = \frac{1}{3}V_{CC}$$

$$T_H = C(R_A + R_B)\ln(2) \approx 0.69C(R_A + R_B)$$

The fall in $v_C$ is given by:

$$v_C = V_{TH}e^{-\frac{t}{CR_B}}$$

$$v_C = V_{TL} = \frac{1}{3}V_{CC} \quad t = T_L \Rightarrow \quad V_{TH} = \frac{2}{3}V_{CC}$$

$$T_L = CR_B\ln(2) \approx 0.69CR_B$$

The total period is:

$$T = T_H + T_L = 0.69C(R_A + 2R_B)$$
BIBLIOGRAPHY


PROBLEMS

Section 12.1: Basic Principles of Sinusoidal Oscillators

12.1. Consider a sinusoidal oscillator consisting of an amplifier having a frequency-independent gain $A$ (where $A$ is positive) and a second-order bandpass filter with a pole frequency $\omega_0$, a pole $Q$ denoted $Q$, and a center-frequency gain $K$.

(a) Find the frequency of oscillation, and find the condition that $A$ and $K$ must satisfy for sustained oscillations.

(b) Derive an expression for $\frac{d\phi}{d\omega}$ evaluated at $\omega = \omega_0$.

(c) Use the result of (b) to find an expression for the per unit change in frequency of oscillation resulting from a phase angle $\Delta \phi$ in the amplifier transfer function.

Hint: $\frac{d}{dx}(\tan^{-1} y) = \frac{1}{1 + y^2} \frac{dy}{dx}$

12.2. For the oscillator described in Problem 12.1, show that independent of the value of $A$ and $K$ the poles of the circuit lie at a radial distance of $\omega_0$. Find the value of $AK$ that results in the poles (a) on the j$\omega$-axis, (b) in the right-half of the s-plane at a horizontal distance from the j$\omega$-axis of $\omega_0/(2Q)$.

12.3. Sketch a circuit for a sinusoidal oscillator formed by an op amp connected in the noninverting configuration and a bandpass filter implemented by an RLC resonator (such as that in Fig. 11.18d). What should the amplifier gain be to obtain sustained oscillations? What is the frequency of oscillation? Find the percentage change in $\omega_0$ resulting from a change of $\pm 1\%$ in the value of (a) $L$, (b) $C$, (c) $R$.

12.4. An oscillator is formed by loading a transconductance amplifier having a positive gain with a parallel RLC circuit and connecting the output to the input directly (thus applying positive feedback with a factor $\beta = 1$). Let the transconductance amplifier have an input resistance of 10 k$\Omega$ and an output resistance of 10 k$\Omega$. The LC resonator has $L = 10 \mu$H, $C = 1000$ pF, and $Q = 100$. For what value of transconductance $G_m$ will the circuit oscillate? At what frequency?

12.5. In a particular oscillator characterized by the structure of Fig. 12.1, the frequency-selective network exhibits a loss of 20 dB and a phase shift of 180° at $\omega_0$. What is the minimum gain and the phase shift that the amplifier must have for oscillations to begin? Consider the circuit of Fig. 12.3(a) with $R_1$ removed so as to realize the comparator function. Find suitable values for all resistors so that the comparator output levels are $\pm 6$ V and so that the slope of the limiting characteristic is 0.1. Use power supply voltages of $\pm 10$ V and assume the voltage drop of a conducting diode to be 0.7 V.
D12.7 Consider the circuit of Fig. 12.3(a) with $R_f$ removed so as to realize the comparator function. Sketch the transfer characteristic. Show that by connecting a dc source $V_B$ to the virtual ground of the op amp through a resistor $R_B$, the transfer characteristic is shifted along the $v_T$-axis to the point $v_T = -(R_f/R_B)V_B$. Utilizing available $\pm 15$ V dc supplies for $\pm V$ and for $V_B$, find suitable component values so that the limiting levels are $\pm 5$ V and the comparator threshold is at $v_T = +5$ V. Neglect the diode voltage drop (that is, assume $V_D = 0$). The input resistance of the comparator is to be $100$ k$\Omega$, and the slope in the limiting regions is to be $\pm 0.05$ V/V. Use standard 5% resistors (see Appendix F).

12.8 Denoting the zener voltages of $Z_1$ and $Z_2$ by $V_{Z1}$ and $V_{Z2}$ and assuming that in the forward direction the voltage drop is approximately 0.7 V, sketch and clearly label the transfer characteristics $v_T - v_T$ of the circuits in Fig. P12.8. Assume the op amps to be ideal.

\[ \text{Fig. P12.8} \]

**Section 12.2: Op-Amp–RC Oscillator Circuits**

12.9 For the Wien-bridge oscillator circuit in Fig. 12.4, show that the transfer function of the feedback network $\left[ \frac{V_a(s)}{V_i(s)} \right]$ is that of a bandpass filter. Find $\omega_0$ and $Q$ of the poles, and find the center-frequency gain.

12.10 For the Wien-bridge oscillator of Fig. 12.4, let the closed-loop amplifier (formed by the op amp and the resistors $R_1$ and $R_2$) exhibit a phase shift of $-0.1$ rad in the neighborhood of $\omega = 1/CR$. Find the frequency at which oscillations can occur in this case, in terms of $CR$. (Hint: Use Eq. 12.11.)

12.11 For the Wien-bridge oscillator of Fig. 12.4, use the expression for loop gain in Eq. (12.10) to find the poles of the closed-loop system. Give the expression for the pole $Q$, and use it to show that to locate the poles in the right half of the s-plane, $R_2/R_1$ must be selected greater than 2.

D'12.12 Reconsider Exercise 12.3 with $R_3$ and $R_4$ increased to reduce the output voltage. What values are required for an output of 10 V peak to peak? What results if $R_3$ and $R_4$ are open circuited?

12.13 For the circuit in Fig. P12.13 find $L(s)$, $L(j\omega)$, the frequency for zero loop-phase, and $R_1/R_2$ for oscillation.

\[ \text{Fig. P12.13} \]

12.14 Repeat Problem 12.13 for the circuit in Fig. P12.14.

*12.15 Consider the circuit of Fig. 12.6 with the 50-k$\Omega$ potentiometer replaced with two fixed resistors: 10 k$\Omega$ between the op amp's negative input and ground, and 18 k$\Omega$. Modeling each diode as a 0.65-V battery in series with a 100-$\Omega$ resistance, find the peak-to-peak amplitude of the output sinusoid.
**12.16** Redesign the circuit of Fig. 12.6 for operation at 10 kHz using the same values of resistance. If at 10 kHz the op amp provides an excess phase shift (lag) of 5.7°, what will be the frequency of oscillation? (Assume that the phase shift introduced by the op amp remains constant for frequencies around 10 kHz.) To restore operation to 10 kHz, what change must be made in the shunt resistor of the Wien bridge? Also, to what must $R_2/R_1$ be changed?

**12.17** For the circuit of Fig. 12.8, connect an additional $R = 10$ kΩ resistor in series with the rightmost capacitor $C$. For this modification (and ignoring the amplitude stabilization circuitry) find the loop gain $A\beta$ by breaking the circuit at node $X$. Find $R_f$ for oscillation to begin, and find $f_o$.

**12.18** For the circuit in Fig. P12.18, break the loop at node $X$ and find the loop gain (working backward for simplicity) to find $V_i$ in terms of $V_o$. For $R = 10$ kΩ, find $C$ and $R_f$ to obtain sinusoidal oscillations at 10 kHz.

**12.19** Consider the quadrature-oscillator circuit of Fig. 12.9 without the limiter. Let the resistance $R_f$ be equal to $2R/(1 + \Delta)$, where $\Delta \ll 1$. Show that the poles of the characteristic equation are in the right-half $s$-plane and given by $s = (1/CR)[(\Delta/4) \pm j]$. (Note: $\Delta = \omega_0^2/C R$)

**12.20** Assuming the diode-clipped waveform in Exercise 12.7 is nearly an ideal square wave and that the resonator $Q$ is 20, provide an estimate of the distortion in the output sine wave by calculating the magnitude (relative to the fundamental) of
(a) the second harmonic,
(b) the third harmonic,
(c) the fifth harmonic,
(d) the root mean square of harmonics to the tenth.

Note that a square wave of amplitude $V$ and frequency $\omega$ is represented by the series

$$\frac{4V}{\pi} \left( \cos \omega t - \frac{1}{3} \cos 3\omega t + \frac{1}{5} \cos 5\omega t - \frac{1}{7} \cos 7\omega t + \cdots \right)$$

**Section 12.3: LC and Crystal Oscillators**

**12.21** Figure P12.21 shows four oscillator circuits of the Colpitts type, complete with bias detail. For each circuit derive an equation governing circuit operation, and find the frequency of oscillation and the gain condition that ensures that oscillations start.

**12.22** Consider the oscillator circuit in Fig. P12.22, and assume for simplicity that $\beta = \infty$.

(a) Find the frequency of oscillation and the minimum value of $R_C$ (in terms of the bias current $I$) for oscillations to start.

(b) If $R_C$ is selected equal to $(1/I)$ kΩ, where $I$ is in milliamperes, convince yourself that oscillations will start. If oscillations grow to the point that $V_o$
is sufficiently large to turn the BJTs on and off, show that the voltage at the collector of $Q_2$ will be a square wave of 1 V peak to peak. Estimate the peak-to-peak amplitude of the output sine wave $V_o$.

**Problem 12.23** Consider the Pierce crystal oscillator of Fig. 12.16 with the crystal as specified in Exercise 12.10. Let $C_1$ be variable in the range 1 to 10 pF, and let $C_2$ be fixed at 10 pF. Find the range over which the oscillation frequency can be tuned. (*Hint:* Use the result in the statement leading to the expression in Eq. 12.27.)
Section 12.4: Bistable Multivibrators

12.24 Consider the bistable circuit of Fig. 12.19(a) with the op amp’s positive input terminal connected to a positive voltage source $V$ through a resistor $R_3$.

(a) Derive expressions for the threshold voltages $V_{TL}$ and $V_{TH}$ in terms of the op amp’s saturation levels $L_+$ and $L_-$, $R_1$, $R_2$, $R_3$, and $V$.

(b) Let $L_+ = -L_- = 13$ V, $V = 15$ V and $R_1 = 10$ k$\Omega$. Find the values of $R_2$ and $R_3$ that result in $V_{TL} = +4.9$ V and $V_{TH} = +5.1$ V.

12.25 Consider the bistable circuit of Fig. 12.20(a) with the op amp’s negative input terminal disconnected from ground and connected to a reference voltage $V_R$.

(a) Derive expressions for the threshold voltages $V_{TL}$ and $V_{TH}$ in terms of the op amp’s saturation levels $L_+$ and $L_-$, $R_1$, $R_2$, and $V_R$.

(b) Let $L_+ = -L_- = V$ and $R_1 = 10$ k$\Omega$. Find $R_2$ and $V_R$ that result in threshold voltages of 0 and $V/10$.

12.26 For the circuit in Fig. 12.26, sketch and label the transfer characteristic $v_o-v_I$. The diodes are assumed to have a constant 0.7-V drop when conducting, and the op amp saturates at ±12 V. What is the maximum diode current?

12.27 Consider the circuit of Fig. 12.26 with $R_1$ eliminated and $R_2$ short-circuited. Sketch and label the transfer characteristic $v_o-v_I$. Assume that the diodes have a constant 0.7-V drop when conducting and that the op amp saturates at ±12 V.

12.28 Consider a bistable circuit having a noninverting transfer characteristic with $L_+ = -L_- = 12$ V, $V_{TL} = -1$ V, and $V_{TH} = +1$ V.

(a) For a 0.5-V-amplitude sine-wave input having zero average, what is the output?

(b) Describe the output if a sinusoid of frequency $f$ and amplitude of 1.1 V is applied at the input. By how much can the average of this sinusoidal input shift before the output becomes a constant value?

D12.29 Design the circuit of Fig. 12.23(a) to realize a transfer characteristic with ±7.5-V output levels and ±7.5-V threshold values. Design so that when $v_I = 0$ V, a current of 0.1 mA flows in the feedback resistor and a current of 1 mA flows through the zener diodes. Assume that the output saturation levels of the op amp are ±12 V. Specify the voltages of the zener diodes and give the values of all resistors.

Section 12.5: Generation of Square and Triangular Waveforms Using Astable Multivibrators

12.30 Find the frequency of oscillation of the circuit in Fig. 12.24(b) for the case $R_1 = 10$ k$\Omega$, $R_2 = 16$ k$\Omega$, $C = 10$ nF, and $R = 62$ k$\Omega$.

D12.31 Augment the astable multivibrator circuit of Fig. 12.24(b) with an output limiter of the type shown in Fig. 12.23(b). Design the circuit to obtain an output square wave with 5-V amplitude and 1-kHz frequency using a 10-nF capacitor C. Use $\beta = 0.462$, and design for a current in the resistive divider approximately equal to the average current in the RC network over $\frac{1}{4}$ cycle. Assuming ±13-V op amp saturation voltages, arrange that the zener operates at a current of 1 mA.
D12.32 Using the scheme of Fig. 12.25, design a circuit that provides square waves of 10 V peak to peak and triangular waves of 10 V peak to peak. The frequency is to be 1 kHz. Implement the bistable circuit with the circuit of Fig. 12.23(b). Use a 0.01-μF capacitor, and specify the values of all resistors and the required zener voltage. Design for a minimum zener current of 1 mA and for a maximum current in the resistive divider of 0.2 mA. Assume that the output saturation levels of the op amps are ±13 V.

D’12.33 The circuit of Fig. P12.33 consists of an inverting bistable multivibrator with an output limiter and a noninverting integrator. (This integrator circuit was studied in Example 2.6.) Using equal values for all resistors except $R_1$ and a 0.5-nF capacitor, design the circuit to obtain a square wave at the output of the bistable multivibrator of 15-V peak-to-peak amplitude and 10-kHz frequency. Sketch and label the waveform at the integrator output. Assuming ±13-V op amp saturation levels, design for a minimum zener current of 1 mA. Specify the zener voltage required, and give the values of all resistors.

Section 12.6: Generation of a Standardized Pulse—The Monostable Multivibrator

*12.34 Figure P12.34 shows a monostable multivibrator circuit. In the stable state, $v_O = L_+, v_A = 0$, and $v_B = -V_{ref}$. The circuit can be triggered by applying a positive input pulse of height greater than $V_{ref}$. For normal operation, $C_1R_1 \ll CR$. Show the resulting waveforms of $v_O$ and $v_A$. Also, show that the pulse generated at the output will have a width $T$ given by

$$T = CR \ln \left( \frac{L_+ - L_-}{V_{ref}} \right)$$

Note that this circuit has the interesting property that the pulse width can be controlled by changing $V_{ref}$.

12.35 For the monostable circuit considered in Exercise 12.19, calculate the recovery time.

D’12.36 Using the circuit of Fig. 12.26, with a nearly ideal op amp for which the saturation levels are ±13 V, design a monostable multivibrator to provide a negative output pulse of 100-μs duration. Use capacitors of 0.1 nF and 1 nF. Wherever possible, choose resistors of 100 kΩ in your design. Diodes have a drop of...
0.7 V. What is the minimum input step size that will ensure triggering? How long does the circuit take to recover to a state in which retriggering is possible with a normal output?

**Section 12.7: Integrated-Circuit Timers**

12.37 Consider the 555 circuit of Fig. 12.27 when the Threshold and the Trigger input terminals are joined together and connected to an input voltage \( v_i \). Verify that the transfer characteristic \( v_o - v_i \) is that of an inverting bistable circuit with thresholds \( V_{TH} = \frac{2}{3}V_{CC} \) and \( V_{TH} = \frac{2}{3}V_{CC} \) and output levels of 0 and \( V_{CC} \).

12.38 (a) Using a 1-nF capacitor \( C \) in the circuit of Fig. 12.28, find the value of \( R \) that results in an output pulse of 10-µs duration.

(b) If the 555 timer used in (a) is powered with \( V_{CC} = 15 \) V, and assuming that \( V_{TH} \) can be varied externally (that is, it need not remain equal to \( \frac{2}{3}V_{CC} \)), find its required value so that the pulse width is increased to 20 µs with other conditions the same as in (a).

12.39 Using a 680-pF capacitor, design the astable circuit of Fig. 12.29(a) to obtain a square wave with a 50-kHz frequency and a 75% duty cycle. Specify the values of \( R_A \) and \( R_B \).

12.40 The node in the 555 timer at which the voltage is \( V_{TH} \) (that is, the inverting input terminal of comparator 1) is usually connected to an external terminal. This allows the user to change \( V_{TH} \) externally (that is, it no longer remains \( \frac{2}{3}V_{CC} \)). Note, however, that whatever the value of \( V_{TH} \) becomes, \( V_{TH} \) always remains \( \frac{2}{3}V_{CC} \).

(a) For the astable circuit of Fig. 12.29, rederive the expressions for \( T_H \) and \( T_L \), expressing them in terms of \( V_{TH} \) and \( V_{CC} \).

(b) For the case \( C = 1 \) nF, \( R_A = 7.2 \) kΩ, \( R_B = 3.6 \) kΩ, and \( V_{CC} = 5 \) V, find the frequency of oscillation and the duty cycle of the resulting square wave when no external voltage is applied to the terminal \( V_{TH} \).

(c) For the design in (b), let a sine-wave signal of a much lower frequency than that found in (b) and of 1-V peak amplitude be capacitively coupled to the circuit node \( V_{TH} \). This signal will cause \( V_{TH} \) to change around its quiescent value of \( \frac{2}{3}V_{CC} \), and thus \( T_H \) will change correspondingly—a modulation process. Find \( T_H \), and find the frequency of oscillation and the duty cycle at the two extreme values of \( V_{TH} \).

**Section 12.8: Nonlinear Waveform-Shaping Circuits**

D*12.41 The two-diode circuit shown in Fig. P12.41 can provide a crude approximation to a sine-wave output when driven by a triangular waveform. To obtain a good approximation, we select the peak of the triangular waveform, \( V \), so that the slope of the desired sine wave at the zero crossings is equal to that of the triangular wave. Also, the value of \( R \) is selected so that when \( v_i \) is at its peak the output voltage is equal to the desired peak of the sine wave. If the diodes exhibit a voltage drop of 0.7 V at 1-mA current, changing at a rate of 0.1-V per decade, find the values of \( V \) and \( R \) that will yield an approximation to a sine waveform of 0.7-V peak amplitude. Then find the angles \( \theta \) (where \( \theta = 90^\circ \) when \( v_i \) is at its peak) at which the output of the circuit is 0.7, 0.65, 0.6, 0.55, 0.5, 0.4, 0.3, 0.2, 0.1, and 0 V. Use the angle values obtained to determine the values of the exact sine wave (that is, 0.7 sin \( \theta \)), and thus find the percent error of this circuit as a sine shaper. Provide your results in tabular form.

![Fig. P12.41](image)

D12.42 Design a two-segment sine-wave shaper using a 10-kΩ input resistor, two diodes, and two clamping voltages. The circuit, fed by a 10-V peak-to-peak triangular wave, should limit the amplitude of the output signal via a 0.7-V diode to a value corresponding to that of a sine wave whose zero-crossing slope matches that of the triangle. What are the clamping voltages you have chosen?

12.43 Show that the output voltage of the circuit in Fig. P12.43 is given by

\[
v_o = -n V_T \ln \left( \frac{v_i}{I_D R} \right), \quad v_i > 0
\]

where \( I_D \) and \( n \) are the diode parameters and \( V_T \) is the thermal voltage. Since the output voltage is propor-
tional to the logarithm of the input voltage, the circuit is known as a logarithmic amplifier. Such amplifiers find application in situations where it is desired to compress the signal range.

12.44 Verify that the circuit in Fig. P12.44 implements the transfer characteristic \( v_O = \frac{v_1 v_2}{v_1 + v_2} \) for \( v_1, v_2 > 0 \). Such a circuit is known as an analog multiplier. Check the circuit’s performance for various combinations of input voltage of values, say, 0.5, 1, 2, and 3 volts. Assume all diodes to be identical, with 700-mV drop at 1-mA current and \( n = 2 \). Note that a squarer can easily be produced using a single input (for example, \( v_1 \) connected via a 0.5-kΩ resistor (rather than the 1-kΩ shown).

**12.45** Detailed analysis of the circuit in Fig. 12.32 shows that optimum performance (as a sine shaper) occurs when the values of \( I \) and \( R \) are selected so that \( RI = 2.5 \, V_T \), where \( V_T \) is the thermal voltage, and the peak amplitude of the input triangular wave is 6.6 \( V_T \). If the output is taken across \( R \) (that is, between the two emitters), find \( v_I \) corresponding to \( v_O = 0.25 \, V_T, \) 0.5 \( V_T, V_T, 1.5 \, V_T, 2 \, V_T, 2.4 \, V_T, \) and 2.42 \( V_T \). Plot \( v_O - v_I \) and compare to the ideal curve given by

\[
v_O = 2.42 \, V_T \sin \left( \frac{v_I}{6.6 \, V_T} \times 90^\circ \right).
\]

Section 12.9: Precision Rectifier Circuits

12.46 Two superdiode circuits connected to a common load resistor and having the same input signal have their diodes reversed, one with cathode to the load, the other with anode to the load. For a sine-wave input of 10 V peak to peak, what is the output waveform? Note that each half cycle of the load current is provided by a separate amplifier, and that while one
amplifier supplies the load current, the other amplifier idles. This idea, called class B operation, is important in the implementation of power amplifiers.

D12.47 The superdiode circuit of Fig. 12.33(a) can be made to have gain by connecting a resistor $R_2$ in place of the short circuit between the cathode of the diode and the negative input terminal of the op amp, and a resistor $R_1$ between the negative input terminal and ground. Design the circuit for a gain of 2. For a 10-V peak-to-peak input sine wave, what is the average output voltage resulting?

D12.48 Provide a design of the inverting precision rectifier shown in Fig. 12.34(a) in which the gain is $-2$ for negative inputs and zero otherwise, and the input resistance is 100 kΩ. What values of $R_1$ and $R_2$ do you choose?

D12.49 Provide a design for a voltmeter circuit similar to the one in Fig. 12.35, which is intended to function at frequencies of 10 Hz and above. It should be calibrated for sine-wave input signals to provide an output of +10 V for an input of 1 V rms. The input resistance should be as high as possible. To extend the bandwidth of operation, keep the gain in the ac part of the circuit reasonably small. As well, the design should be such as to reduce the size of the capacitor $C$ required. The largest value of resistor available is 1 MΩ.

12.50 Plot the transfer characteristic of the circuit in Fig. P12.50.

Fig. P12.50

12.51 Plot the transfer characteristics $v_{o1} - v_i$ and $v_{o2} - v_i$ of the circuit in Fig. P12.51.

Fig. P12.51

D12.52 Sketch the transfer characteristics of the circuit in Fig. P12.52.

Fig. P12.52

D12.53 A circuit related to that in Fig. 12.38 is to be used to provide a current proportional to $v_A(v_A > 0)$ to a light-emitting diode (LED). The value of the current is to be independent of the diode’s nonlinearities and variability. Indicate how this may be done easily.

D12.54 In the precision rectifier of Fig. 12.38, the resistor $R$ is replaced by a capacitor $C$. What happens? For equivalent performance with a sine-wave input of 60-Hz frequency with $R = 1$ kΩ, what value of $C$ should be used? What is the response of the modified circuit at 120 Hz? at 180 Hz? If the amplitude of $v_A$ is kept fixed, what new function does this circuit perform? Now consider the effect of a waveform change on both circuits (the one with $R$ and the one with $C$).
For a triangular-wave input of 60-Hz frequency that produces an average meter current of 1 mA in the circuit with $R$, what does the average meter current become when $R$ is replaced with the $C$ whose value was calculated above?

**12.55** A positive-peak rectifier utilizing a fast op amp and a junction diode in a superdiode configuration, and a 10-$\mu$F capacitor initially uncharged, is driven by a series of 10-V pulses of 10-$\mu$s duration. If the maximum output current that the op amp can supply is 10 mA, what is the voltage on the capacitor following one pulse? two pulses? ten pulses? How many pulses are required to reach 0.5 V? 1.0 V? 2.0 V?

**D12.56** Consider the buffered precision peak rectifier shown in Fig. 12.40 when connected to a triangular input of 1-V peak-to-peak amplitude and 1000-Hz frequency. It utilizes an op amp whose bias current (directed into $A_2$) is 10 nA and diodes whose reverse leakage current is 1 nA. What is the smallest capacitor that can be used to guarantee an output ripple less than 1%?