Introduction to Continuum Damage Mechanics

By:
Dr. Curtis F. Berthelot P.Eng.
Department of Civil and Geological Engineering
Centre of Excellence for Transportation and Infrastructure

Engineering Modeling Frameworks

- Engineers require a methodology that accurately predicts (models) whole life performance of the system.
- Three basic-modeling techniques:
  - Purely-Empirical (Experience based)
  - Phenomenological-Empirical (fudge tests that make us feel better, but don’t really tell us anything conclusive)
  - Mechanistic-based (future)
Empiricism is defined as:
- “Relying on experience or observation without due regard for system and/or theory.”

Employs observed field performance to derive performance based design relationships.
- Classical statistical regression analysis to derive road performance relations based solely on judgment inferred from road performance observations.
- Engineer’s intuition-dangerous in a litigious society.
Phenomenological-Empirical Modeling

- Phenomenological materials tests are used to simulate field conditions and performance.
  - Make us feel better, but do not really tell us anything of engineering significance.
  - Often used to “rank” alternatives in terms of performance.
  - May give us the wrong answer or prioritization scheme.
- Phenomenological tests do not provide mechanistic measures of behavior that can be used in a mechanistic modeling framework.

Limitations of Phenomenological Empirical Models

- Modern engineered systems are comprised of diverse structures that are subjected to variable loadings.
- Significant models and supporting databases are required to empirically predict performance.
- Traditional phenomenological material tests provide test dependent material behavior indicators that do not characterize the fundamental thermomechanical constitutive behavior of the materials used to construct the system.
Limitations of Phenomenological Empirical Models

- Performance predictions based exclusively on judgment and/or experience may be biased and/or inaccurate (usually both).
- Changing field state conditions; ongoing developments in engineering, materials, system functionality requirements, construction methods etc, are beyond the inference of traditional performance models.
- As an engineer looking at the next 30 years of your career, begin to develop skills and knowledge that establish a more reliable check and balance system for your engineer’s intuition.

Scientific-Engineering Process

1. Observe
2. Measure
3. Model
4. Validate
5. Economic Evaluation.
Mechanistic Modeling

- To overcome the limitations of purely-empirical and phenomenological-empirical techniques, mechanistic-modeling methods that draw upon materials science, engineering thermomechanics, and computational capabilities are being used:
  - Mechanistic-Empirical
  - Purely Mechanistic (closed form solutions-ideal).
  - Damage mechanics:
    - Flow
    - Fracture

Mechanistic Modeling

- Continuum Mechanics:
  - Modeling materials/systems as homogeneous matter (ignores internal cracks, voids, asperities and anomalies at link scales much smaller than the boundary geometry).
  - Encodes mechanical material behavior with constitutive relationships (i.e. stress & strain).
- Micro Mechanics:
  - Study of materials and systems at an atomic or inter-particulate level.
Mechanistic Modeling

- Mechanics is the application of the principles of thermodynamics to characterize the mechanical behavior of materials and structures:
  - Based on Natural Laws:
    - First and Second Laws of Thermodynamics.
    - Conservation of linear and angular momentum.
    - Conservation of charge
  - Powerful and reliable framework for engineers

Galileo Galilei

- 1564-1642
- Originally a professor of medicine at Pisa.
- Focused on mathematics (work of Euclid and Archimedes).
- Later became professor of mathematics and focused on falling bodies (dynamics).
- Galileo went against many theories of Aristotle, however his new science drew lectures of over 2000 students.
- Worked in strength of materials for ship building industry.
Santa Maria del Fiore - 1296 to 1436

Robert Hooke

- 1635-1703
- Curator of experiments at Royal Society.
- Performed many experiments evaluating the strength and performance of common materials.
- 1660 – Ceiiinossstttuv-“ut tensio sic vis”
  - “As the load so does the displacement”
- Hooke’s Model (called Law) established the foundation of observed linear elastic behaviour.
Sir Isaac Newton

- 1642-1727 (Born on Xmas day of the year that Galileo died)
- 1665 – Apple fell on Newton’s head in Hampton Court, Woolsthorpe, England (natural gravity).
- Studied work of Galileo, Copernicus and Keppler and derived 3 axioms of nature to describe the natural laws.
  - Law I: Every body remains in a state of rest or steady motion in a straight line unless acted on by a force.
  - Law II: The quantity of altered motion is proportional to the force acting upon it (F=ma).

Mechanistic Road Materials Characterization and Road Structural Analysis

- Law III: For equilibrium to exist, every action must have an equal and opposite action for equilibrium to exist, i.e. Conservation of Linear Momentum.
- Newton had to develop temporal calculus to provide time wise derivatives of velocity to give acceleration which was related to the force required to move mass ‘a’.

\[ F = ma \]

\[ a = \frac{dV}{dt} = \frac{d^2 s}{dt^2} \]
Sir Isaac Newton

- These laws were published in 1686, “Philosophiae Naturalis Principia Mathematica.”
  
  “If it is I that has seen farthest, it is because I stood on the shoulders of giants.”

Leonard Euler

- 1707-1783
- Father of Strength of Materials.
- Most regarded 18th Century scientist.
- Studied under John Bernoulli.
- Worked at Russian Academy of Science in St. Petersburg when it opened in 1725.
- Jacob and Daniel Bernoulli worked with Euler.
- Wrote over 400 papers and textbooks; 200 of which were during last 20 years of his life when he was blind.
Kinematics: Strain - Displacement Relations

- Leonard Euler (1707-1783).
- Engineering Uniaxial Strain ($\varepsilon_{xx}$) is the square of the deformed length minus the square of the initial length divided by the square of the initial length:
  \[
  \varepsilon_{xx} = \lim_{\Delta l \to 0} \frac{1}{2} \left( \frac{|l_f|^2 - |l_i|^2}{|l_i|^2} \right)
  \]

- For infinitesimal deformations, uniaxial strain may be approximated by:
  \[
  \varepsilon_{xx} \approx \frac{l_f - l_i}{l_i} \frac{\Delta l}{l_i}
  \]

Infinitesimal Strain - Displacement Relation

- Normal and shear strains:
  \[
  \begin{align*}
  \varepsilon_{xx} &= \frac{\partial u}{\partial x} \quad &\varepsilon_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\
  \varepsilon_{yy} &= \frac{\partial v}{\partial y} \quad &\varepsilon_{yz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\
  \varepsilon_{zz} &= \frac{\partial w}{\partial z} \quad &\varepsilon_{xz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}
  \end{align*}
  \]

  \rightarrow 6 unknowns
Conservation Laws

- Conservation of mass (trivial):
  \[ \frac{d}{dt} \int \rho dV = 0 \Rightarrow \frac{d\rho}{dt} + \rho \frac{du}{dt} = 0 \]

- Conservation of linear momentum:
  \[ \sum F = 0 \Rightarrow \sigma_{ij} = \rho f_i - \rho d^2 u_j dt^2 = 0 \]

- Conservation of angular momentum:
  \[ \sum M = 0 \Rightarrow \sigma_{ij} = \sigma_{ji} \]

- Conservation of charge for C.E. (Trivial)
Conservation Laws

- Conservation of Energy (1st Law of Thermodynamics):
  \[ \frac{dU}{dt} = \frac{dh}{dt} + \frac{dw}{dt} \Rightarrow \rho \frac{dU}{dt} = \sigma_{ij} e_{ij} - \dot{q}_{ij} + \rho r \]

- Entropy Inequality (2nd Law of Thermodynamics):
  \[ \rho \frac{dS}{dt} + \left( \frac{q_i}{T} \right)_i - \frac{\rho r}{T} \geq 0 \]

- Given St. Venants rules, the field equations are significantly reduced:
  - Conservation of Energy
    \[ \sigma_{ij} e_{ij} - \dot{q}_{ij} + \rho r = \rho \frac{dU}{dt} \Rightarrow \sigma_{ij} e_{ij} = \rho \frac{dU}{dt} \]
  - Entropy Inequality:
    \[ \rho \frac{dS}{dt} + \left( \frac{q_i}{T} \right)_i - \frac{\rho r}{T} \geq 0 \Rightarrow \rho \frac{dS}{dt} \geq 0 \]
  - Conservation of Linear Momentum:
    \[ \sigma_{j,j} + \rho f_j - \rho \frac{d^2 u_j}{dt^2} = 0 \Rightarrow \sigma_{j,j} = 0 \]
**Engineering Properties of Materials**

- **Thermomechanical Approach**
  - Applicable to all engineered systems (multidisciplinary approach)
  - Natural laws do not vary in time, therefore, thermomechanical models provide a stable platform for future model improvements
  - Thermomechanical material characterization provides direct mapping to primary response and performance of the material in the field
  - Thermomechanics can be applied as a uniform theory to all materials and systems

---

**Cauchy’s Uniqueness Theorem of Stress**

- Augustin Louis Cauchy, 1789-1857.
- Cauchy applied his knowledge as a hydrodynamics engineer to formulate force over unit area for strength of solid materials purposes.
Cauchy’s Uniqueness Theorem of Stress

Conservation Laws: Kinetics

Differential Equations of Equilibrium

- Conservation of linear momentum must hold for every point in the material body.

\[
\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} + \frac{\partial \sigma_{31}}{\partial x_3} + X_1 = 0
\]

\[
\frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{32}}{\partial x_3} + X_2 = 0 \Rightarrow \sigma_{i,k} + X_i = 0 \text{ in } \mathbf{V}
\]

\[
\frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} + X_3 = 0
\]
Barre St. Venant

- 1797-1886
- Father of elasticity.
- Derived compatibility equations between deformations and strain.
- Derived 7 rules for characterizing deformation of materials.

1. Uniform stress-strain field in sample: St. Venant’s principle of sample geometry.
2. Principle of Link scale of asperities: Specimen size must be an order of magnitude larger than maximum asperity size contained in the material:
   - Very important for particulate composite road materials
3. Eliminate thermal gradients in specimen (test at constant temperature: DT=0)
4. Applied load rate should be much slower than natural frequency of material (quasi-static):
\[ f_{\text{steel}} \approx 10,000 \text{ Hz}; \quad f_{\text{humans}} \approx 2 \text{ Hz} \]

5. Eliminate body forces (creep) in specimen:
\[ f_{\text{body}} \ll f_{\text{applied}} \]

6. Eliminate heat sources in specimen:
\[ r = 0 \]

7. Eliminate inertial effects:
\[ \frac{\partial^2 u_i}{\partial t^2} = 0 \]

---

**Damage Mechanics**

**Structural Evaluation**

- Performance Based Whole Life Load Equivalencies
- Probabilistic-Performance Based Whole Life User Cost/Benefit Analysis
- Loading
- Climatic Conditions
- Structural Geometry
- Structure Materials
- Thermo-Mechanistic Material Constitutive Characterization
  - Resilience
  - Viscoplastic Creep
  - Fracture
  - Healing
- Primary Structure Responses (Finite Elements)
  - \( \mu = \mu(\dot{x}, t) \)
  - \( \sigma = \sigma(\dot{x}, t) \)
  - \( \varepsilon = \varepsilon(\dot{x}, t) \)
  - \( T = T(\dot{x}, t) \)
- Performance Prediction Damage Model
  - Resilience
  - Viscoplastic Creep
  - Viscoplastic Fracture
  - Autogeneous Healing

- Performance Based Whole Life Structure Equivalencies
- Probabilistic-Performance Based Whole Life Agency Cost/Benefit Analysis

---

**Introduction to Continuum Mechanics**

34
Other Material Properties:
- Physical: Density, Color, Texture, Size
- Chemical
- Electrical
- Thermal
- Optical
- Magnetic

Material Constitutive Relations:
- Strength
- Stiffness
- Hardness
- Ductility
- Durability (In Field)
- Creep/Flow
- Fracture
- Elastic/resilient
- Viscoplasticity
- Corrosion/wear
Mechanistic Material Modeling Process

- **Kinetic Equilibrium:**
  - Branch of mechanics dealing with motion of the material body with reference to forces acting on the body (conservation of momentum)

- **Kinematic Relationships:**
  - Branch of mechanics dealing with motion of the material body without reference to forces acting on the body (strain-displacement relations)

- **Material Constitutive Relations:**
  - Material specific relationships between kinetics and kinematics constrained by the natural laws of thermodynamics

Concept of Stress

- Pivotal point in materials science and engineering was the realization that the strength of a material is a function of the concentration of loads (stress) and displacements (strain).

- D’Vinci and Galileo: conducted axial strength tests of wires varying in length (strength of longer wires was found to be less than short wires on average-why?).

- Galileo: “Two New Sciences” 1631-1642 first complete documentation of the scientific method to evaluate strength of materials.
**Kinetic Equilibrium**

- Conservation of angular momentum is satisfied throughout the body.

\[
\begin{align*}
\sigma_{23} &= \sigma_{32} \\
\sigma_{13} &= \sigma_{31} \Rightarrow \sigma_{ij} &= \sigma_{ji} \text{ in } V \\
\sigma_{12} &= \sigma_{21}
\end{align*}
\]

**Kinematics: Strain - Displacement Relations**

- In matrix form

\[
\begin{bmatrix}
\varepsilon_x & \varepsilon_y & \varepsilon_z & \gamma_{yz} & \gamma_{zx} & \gamma_{xy}
\end{bmatrix}
\]

- Three special cases of the state of strain at a material point are mentionable.
CASE I: Generalized Plane Strain. If all the components of shear strain that have subscripts in one coordinate direction are negligible, then the material point is said to be in a state of generalized plane strain. Thus, for example

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yy} & \varepsilon_{zz} \\ \varepsilon_{yy} & \varepsilon_{xx} & 0 \\ \varepsilon_{zz} & 0 & 0 \end{bmatrix}$$

CASE II: Plane Strain

If all the components of strain lie in a single plane, then the material point is said to be in a state of plane strain. Thus, for example

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yy} & 0 \\ \varepsilon_{yy} & \varepsilon_{xx} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
CASE III: Uniaxial Strain
If all components of strain are negligible except for one component of normal strain, then the material point is said to be in a state of uniaxial strain. Thus, for example

\[ \varepsilon = [\varepsilon_{xx} \ 0 \ 0 \ 0] \]
Accountability

Not a very popular word these days. Few people like the idea of taking full responsibility for their actions. However, without personal and corporate accountability, we run the very real risk of losing everything we have worked so hard to gain. Each person needs to carry a part of the load. We believe in and practice accountability, so that our actions match our words.