1 Servo System using a Armature Controlled DC Servo Motor

1.1 Principles of operation

A DC motor is used in a control system where an appreciable amount of shaft power is required. The DC motors are either armature-controlled with fixed field, or field-controlled with fixed armature current. DC motors used in instrument employ a fixed permanent-magnet field, and the control signal is applied to the armature terminals.

Fig. 1. (a) Schematic diagram of an armature-controlled DC servo motor, (b) Block diagram

In order to model the DC servo motor shown in Fig. 1, we define parameters and variables as follows.

- \( R_a \) = armature-winding resistance, ohms
- \( L_a \) = armature-winding inductance, henrys
- \( i_a \) = armature-winding current, amperes
- \( i_f \) = field current, amperes
- \( e_a \) = applied armature voltage, volts
- \( e_b \) = back emf, volts
- \( \theta \) = angular displacement of the motor shaft, radians
- \( T \) = torque delivered by the motor, lb-ft
- \( J \) = moment of inertia of the motor and load referred to the motor shaft, slug-ft\(^2\).
- \( f \) = viscous-friction coefficient of the motor and load referred to the motor shaft, lb-ft/rad/sec

The torque \( T \) is delivered by the motor is proportional to the product of the armature current \( i_a \) and the air gap flux \( \Psi \), which is in turn is proportional to the field current

\[
\Psi = K_f i_f
\]
where $K_f$ is a constant. The torque can be written as

$$T = K_f i_a$$

where $K_a$ is also constant. Therefore, the torque is proportional to the armature current so that with a motor torque constant $K$,

$$T = K i_a$$

The back emf is proportional to the angular velocity $d\theta/dt$. Thus, with a back emf constant $K_b$, we have

$$e_b = K_b \frac{d\theta}{dt}$$

The speed of an armature controlled DC servo motor is controlled by the armature voltage $e_a$, which is supplied by a power supply (or amplifier). The differential equation for the armature circuit is

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

The armature current produces the torque which is applied to the inertia and friction.

$$J \frac{d^2 \theta}{dt^2} + f \frac{d\theta}{dt} = T = K i_a$$

Assuming that all initial condition are zero, and taking the Laplace transforms of the above three differential equations, we obtain the following equations in the Laplace transform.

$$K_b s \Theta(s) = E_b(s)$$

$$(L_a s + R_a) I_a(s) + E_b(s) = E_a(s)$$

$$(J s^2 + f s) \Theta(s) = T(s) = K I_a(s)$$

Considering $E_a(s)$ as the input, and $\Theta(s)$ as the output, we can construct the block diagram shown in Fig. 1 (b) from these three equations. The effect of the back emf is seen to be the feedback signal proportional to the speed of the motor. This back emf thus increases the effective damping of the system.

The transfer function of this system is obtained as follows.

$$\frac{\Theta(s)}{E_a(s)} = \frac{K}{s[L_a J s^2 + (L_a f + R_a J)s + R_a f + KK_b]}$$

The inductance $L_a$ in the armature circuit is usually small and maybe neglected. If $L_a$ is neglected, the transfer function is reduced to

$$\frac{\Theta(s)}{E_a(s)} = \frac{K}{s[(L_a f + R_a J)s + R_a f + KK_b]} = \frac{K_m}{s(T_m s + 1)}$$

where

$$K_m = \frac{K}{R_a f + KK_b} = \text{motor gain constant}$$

$$T_m = \frac{R_a J}{R_a f + KK_b} = \text{motor time constant}$$

Recalling that the angular velocity is the derivative of the angular position,

$$\omega = \frac{d\theta}{dt} \quad \text{or} \quad \Omega(s) = s\Theta(s)$$
we have transfer function from the input \( E_a(s) \) to the angular velocity \( \Omega(s) \),

\[
\frac{\Omega(s)}{E_a(s)} = \frac{K_m}{(T_m s + 1)}
\]

Applying the final value theorem to the response to the unit step input of \( 1 \text{V}, 1/s \),

\[
\omega(\infty) = \lim_{s \to 0} s \frac{K_m}{(T_m s + 1)} \frac{1}{s} = K_m
\]

The gain \( K_m \) thus means the final angular velocity that the DC motor reaches with the input voltage of \( 1 \text{V} \). \( T_m \) is the time constant that indicates time for the angular velocity to reach \( 1 - e^{-1} = 0.6321 \). Therefore, these constants \( K_m \) and \( T_m \) can be measured without knowing the mechanical parameters \( J, f \) and the torque delivered by the DC motor.

![Diagram](image)

Fig. 2. (a) Positional Servomechanism

1.2 Analog Circuit

Fig. 2 is the diagram showing the positional servomechanism applied to a DC servo motor. A rotary position sensor (potentiometer - right) is coupled to the motor shaft to measure the angle. This measured angular position is compared to the reference voltage set by another potentiometer (left) to achieve negative feedback to bring the motor’s angular position to the same position as set by the reference potentiometer. Our design project is to accomplish the negative feedback by the microprocessor 68HC11. We do not have the load connected to the DC servo motor. But, when an external load of \( J_L \) and \( f_L \) is connected to the output shaft of the down gear of gear ratio \( n \), both moment of inertia \( J \) and friction coefficient \( f \) are modified as

\[
J = J_m + n^2 J_L
\]

\[
f = f_m + n^2 f_L
\]

changing the mechanical system's time constant \( T_m \) and gain \( K_m \).

We use a small DC motor to realize the positional servo mechanism. Before we do computer control by the microcontroller, we build a stand-alone positional servo system from analog components. The DC motor is Servo ST47 (HobbyWorld) modified to give continuous rotation without stoppers that limit rotation to \( \pm 90^\circ \). Torque at 6 V is 50 oz/in, and speed at 6 V is 0.19 sec/60\(^\circ\) according to the spec. on the box. The schematic diagram of the servo system is shown in Fig. 3. Major components are,

1. Servo ST47 (HobbyWorld) modified - this has a built-in rotary potentiometer 10KΩ.
2. Two of LM675 (National Semiconductor) 3A Power OP. Amp.
3. Wall plug-in type DC power supply, 12V and 1A or larger

Since this servo system is a linear servo system which requires bipolar error signal for the negative feedback, we need a bipolar power supply that can supply ±6 V. Use one power OP. Amp. LM675 as a split power supply of ±6 V from a single 12 V DC power supply. The other OP. Amp. LM675 is used as an inverting power amplifier of Gain -1 to drive the DC servo motor to the full voltage of 6V. Attach some kind of position indicator (needle pointer) so we know the position of the motor’s output shaft. The potentiometer VR1 set a reference angle. So, as you move the pot VR1, then the position indicator should also move following the movement of VR1.

![Diagram of Positional Servomechanism](image)

**Fig. 3.** (a) Positional Servomechanism, implementation

1.3 Preparatory Measurement

The constructed DC servo system is represented by the block diagrams shown in Fig. 4. We assume that potentiometers both reference and built-in the DC motor are connected to the supply voltage of ±6 V as shown in the schematic diagram. Another important assumption is that the angle is measured in terms of “revolutions” instead of radians or degrees. So, 90° is 0.25 revolutions. This also make the velocity to be measured in “rps” revolutions per second. Once the stand-alone analog circuit for the positional servo system is built, make sure first that the output angle $\theta$ follows the movement of VR1 potentiometer. In order to make the block diagram to represent exactly the constructed system, measure $K_m$ and $T_m$.

1. **$K_m$ Open loop gain** Disconnect feedback to DAC terminal of R6 and potentiometer VR1 to AD0 terminal of R7. Apply 1 V to DAC terminal to turn the motor continuously. Observe the output signal from the position sensor potentiometer VR2 with a scope. Calculate the velocity in terms “revolutions per second, rps” which is $K_m$. 


2. **$T_m$ Open loop time constant**  Connect a function generator to ADC terminal, then feed a rectangular wave of $\pm 1$ V (0.2 - 0.5 Hz). Observe the output signal from the VR2 with a scope. From the exponential rise or decay part of the waveform, measure the open loop time constant $T$ from the function $e^{-t/T}$. Actually, measure the time $T$ that the amplitude decays down to $1 - e^{-1} = 0.6321$.

When we have a unity gain feedback around a forward transfer function $G(s)$, the feedback system's transfer function is given by

$$G_c(s) = \frac{G(s)}{1 + G(s)}$$

Since our $G(s)$ is

$$G(s) = \frac{12K_m}{s(T_m s + 1)}$$

The closed loop transfer function $G_c(s)$ is given by

$$G_c(s) = \frac{12K_m}{T_m s^2 + s + 12K_m}$$

From the characteristic equation

$$T_m s^2 + s + 12K_m = T_m(s + a)(s + b) = 0$$

we will likely have two real roots $s = -a$ and $s = -b$. From the time response,

$$C_1e^{-at} + C_2e^{-bt} = C_1e^{-\frac{t}{T_1}} + C_2e^{-\frac{t}{T_2}}$$

We can calculate two time constants $T_1 = \frac{1}{a}$ and $T_2 = \frac{1}{b}$. The expected closed loop time constant, representing how fast the output reaches the reference angle, is close to the larger time constant of the two, $T_1$ and $T_2$.

Fig. 4. (a) Positional Servomechanism, block diagram
2 Design Project

2.1 Stand-alone Analog Position Servo System

We use a small DC motor to realize the positional servo mechanism. Before we do computer control by the microcontroller, we build a stand-alone positional servo system from analog components. The DC motor is Servo ST47 (HobbyWorld) modified to give continuous rotation without stoppers that limit rotation to ±90°. Torque at 6 V is 50 oz/in, and speed at 6 V is 0.19 sec/60° according to the spec. on the box. The schematic diagram of the servo system is shown in Fig. 3. Major components are,

1. Servo ST47 (HobbyWorld) modified - this has a built-in rotary potentiometer 10KΩ.

2. Two of LM675 (National Semiconductor) 3A Power OP. Amp.

3. Wall plug-in type DC power supply, 12V and 1A or larger

Since this servo system is a linear servo system which requires bipolar error signal for the negative feedback, we need a bipolar power supply that can supply ±6 V. Use one power OP. Amp. LM675 as a split power supply of ±6 V from a single 12 V DC power supply. The other OP. Amp. LM675 is used as an inverting power amplifier of Gain -1 to drive the DC servo motor to the full voltage of 6V. Attach some kind of position indicator (needle pointer) so we know the position of the motor's output shaft. The potentiometer VR1 set a reference angle. So, as you move the pot VR1, then the position indicator should also move following the movement of VR1.

Fig. 3. (a) Positional Servomechanism, implementation
2.2 Computer Control for the Position Servo System

Negative feedback has already been implemented in the analog position servo system. The angular position measured in terms of revolutions is measured by a rotary potentiometer VR2 embedded in the DC servo motor ST47 as shown in Fig. 3. The angle is converted to voltage with a gain (or scale factor) of 12 volts/revolution. This voltage is compared with the reference voltage from the position setting potentiometer VR1. The difference is then negatively fed to the power amplifier of gain 1 that applies armature voltage to the servo motor. This negative feedback is shown in the block diagram in Fig. 4. When we form a unity gain feedback around a forward transfer function $G(s)$, the feedback system's transfer function is given by

$$G_c(s) = \frac{G(s)}{1 + G(s)}$$

Since our $G(s)$ is

$$G(s) = \frac{12K_m}{s(T_m s + 1)}$$

The closed loop transfer function $G_c(s)$ is given by

$$G_c(s) = \frac{12K_m}{T_m s^2 + s + 12K_m}$$

![Fig. 4. (a) Positional Servomechanism, block diagram](image)

Our design project is to replace this analog negative feedback with digital negative feedback controller. Digital controller can provide more flexible controller design such as PID (Proportional-Integral-Differential) controller by means of software implementation. 68HC11E9 has a built-in 4 channel ADC and a timer to control the sampling interval, but there is no DAC. So, we interface a 8 bit DAC to 68HC11 using port B and port C. Upon completing these hardware setups, we connect AD0, AD1 and DAC labeled in Fig. 3 to the ADC inputs and DAC output. Then, calculate difference between the reference and the actual voltage value proportional to the angular position. A gain may be multiplied to the difference if desired. Since the forward gain depends on the supply voltage which determines the fastest possible speed, the gain adjustment greater than 1 is meaningless. The gain (multiplier) for the difference signal must be less than 1. One useful aspect of the digital controller is to make correction to the angle measurement when the rotary potentiometer crosses the gap of its variable resistance. When the rotary potentiometer goes beyond ±180° past the extreme end of the potentiometer, the voltage at the brush...
jumps from +6 V to -6 V or vice versa. We can build intelligence to detect when the potentiometer gap is encountered and calculate the correct angle beyond the boundary.

**Design Objective**

In summary, the objective of EE331 design is to design and implement a digital controller for the DC positional servo system. To accomplish this objective, we consider the following design steps (components).

1. Build the analog DC positional servo system given in Fig. 3, and make a functional servo system ready to apply digital control.
2. Learn how to use port B and port C to interface an external device to 68HC11.
3. Interface a 8 bit DAC (Digital to Analog Converter) using port B and/or port C.
4. Develop codes to output a wave form to DAC, including conversion from signed integer to offset binary used in DAC.
5. Learn how to use built-in ADC to acquire sampled data from two channels, one for the reference and the other for the actual angular position.
6. Realize successive approximation ADC by using the interface DAC to understand how ADC works (will not be used in the design, though).
7. Learn how to set the sampling interval with built-in timer in two different ways, polling and interrupt.
8. Integration of the developed software modules to realize the digital controller.